FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2022 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: **(i)** Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed**.

Q. No. 1. **(a)** Let u=[y, z, x] and v=[yz, zx, xy], f = xyz and g = x+ y+ z. Find div (grad (*fg*)). **(10)**

(b) Evaluate $\int_C F(r).dr$ counter clockwise around the boundary C of the region R by Green's theorem, where **(10)**

$$F = [y, -x], C \text{ the circle } x^2 + y^2 = 1/4$$

Q. No. 2. **(a)** Three forces P, Q, R, acting at a point, are in equilibrium, and the angle between P and Q is double of the angle between P and R. Prove that $R^2 = Q(Q - P)$. **(10)**

(b) Find the centre of mass of a semi-circular lamina of radius a whose density varies as the square of the distance from the centre. **(10)**

Q. No. 3. **(a)** A particle moves in such a way that its position vector at time t is **(10)**

$$r = (a\cos nt)\mathrm{i} + (b\sin nt)\mathrm{j},$$

Where a, b, n are constants and a>b>0. Show that the path of the particle is an ellipse of semi-major and minor axes a, b respectively, and that the field of force is directed towards the centre of the ellipse. Also find the maximum speed.

(b) An aeroplane is flying with uniform speed v_0 in an arc of a vertical circle of radius a, whose centre is at a height h vertically above a point O of the ground . If a bomb is dropped from the aeroplane when at a height Y and strikes the ground at O, show that Y satisfies the equation **(10)**

$$KY^2 + Y(a^2 - 2hK) + K(h^2 - a^2) = 0,$$

Where $K = h + \frac{ga^2}{2{v_0}^2}$.

Q. No.4. **(a)** Solve the given initial-value problem. Give the largest interval *I* over which the solution is defined. **(10)**

$$xy' + y = e^x, \ y(1) = 2.$$

(b) Find the general solution of the given higher-order differential equation. **(10)**

$$y''' - 4y'' - 5y' = 0$$

Q. No. 5. **(a)** Find two power series solutions of the given differential equation about the ordinary point x=0. **(10)**

$$y'' - 2xy' + y = 0.$$

(b) Find the general solution of the given Bessel's equation on (0, ∞). **(10)**

$$x^2y'' + xy' + (9x^2 - 4)y = 0$$

Q. No. 6. **(a)** Find the Fourier series of the given function f(x), which is assumed to have the period 2π. Show the details of your work. **(10)**

$$f(x) = \begin{cases} x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$$

(b) Find u(x,t) for the string of length L=1 and $c^2=1$ when the initial velocity is zero and the initial deflection with small k (say, 0.01) is $kx(1-x)$. **(10)**

Q. No. 7. **(a)** Use the Bisection method to determine an approximation to the root of the given function in the interval [1,2] that is accurate to at least within 10^{-4}. **(10)**

$$f(x) = x^3 + 4x^2 - 10 = 0.$$

(b) Values for $f(x) = xe^x$ are given in the following table. Use all the applicable three-point and five-point formulas to approximate $f'(2.0)$. **(10)**

x	1.8	1.9	2.0	2.1	2.2
f(x)	10.889365	12.703199	14.778112	17.148957	19.85503

Q. No. 8. **(a)** Use the Modified Euler method to approximate the solution to each of the following initial-value problem, **(10)**

$$y' = -5y + 5t^2 + 2t, \quad 0 \le t \le 1, \qquad y(0) = \frac{1}{3}, with\ h = 0.1$$

(b) Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on [1, 2]. Use $p_0 = 1$. **(10)**

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