Class 9 Mathematics FBISE
Solved Paper 2022

Section B

Q2.

(i) **Find the values of x and y if** $-3\begin{bmatrix}1 & -2\\ -3 & x\end{bmatrix} + 2\begin{bmatrix}2 & -y\\ -1 & 2\end{bmatrix} = \begin{bmatrix}1 & 2\\ 7 & -2\end{bmatrix}$

Sol: $-3\begin{bmatrix}1 & -2\\ -3 & x\end{bmatrix} + 2\begin{bmatrix}2 & -y\\ -1 & 2\end{bmatrix} = \begin{bmatrix}1 & 2\\ 7 & -2\end{bmatrix}$

$\begin{bmatrix}-3 & 6\\ 9 & -3x\end{bmatrix} + \begin{bmatrix}4 & -2y\\ -2 & 4\end{bmatrix} = \begin{bmatrix}1 & 2\\ 7 & -2\end{bmatrix}$

$\begin{bmatrix}-3+4 & 6-2y\\ 9-2 & -3x+4\end{bmatrix} = \begin{bmatrix}1 & 2\\ 7 & -2\end{bmatrix}$

$\begin{bmatrix}1 & 6-2y\\ 7 & -3x+4\end{bmatrix} = \begin{bmatrix}1 & 2\\ 7 & -2\end{bmatrix}$

Equating corresponding elements on both sides, we get

$6 - 2y = 2$ and $-3x + 4 = -2$

$-2y = 2 - 6$ $\quad$ $-3x = -2 - 4$

$-2y = -4$ $\quad$ $-3x = -6$

$y = \frac{-4}{-2}$ $\quad$ $x = \frac{-6}{-3}$

$\boldsymbol{y = 2}$ **Ans** $\quad$ $\boldsymbol{x = 2}$ **Ans**

(ii) **Simplify** $\frac{3+2i}{3+i}$

Sol: To simplify this we need to rationalize the denominator by multiplying and dividing by conjugate of the denominator $(\mathbf{3-i})$.

$$\frac{3+2i}{3+i} = \frac{3+2i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{3+2i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{(3+2i)(3-i)}{(3+i)(3-i)}$$

$$= \frac{9-3i+6i-2i^2}{(3)^2-(i)^2}$$

$$= \frac{9+3i-2(-1)}{9-(-1)}$$

$$= \frac{9+3i+2}{9+1}$$

$$= \frac{11+3i}{10}$$

$$\frac{3+2i}{3+i} = \frac{11}{10} + \frac{3i}{10}$$ **Ans**

(iii) Simplify $\frac{x^{p(q-r)}}{x^{q(p-r)}} \div \left(\frac{x^q}{x^p}\right)^r$

Sol: $\frac{x^{p(q-r)}}{x^{q(p-r)}} \div \left(\frac{x^q}{x^p}\right)^r = \frac{x^{pq-pr}}{x^{pq-qr}} \div \frac{x^{qr}}{x^{pr}}$

$$= \frac{x^{pq-pr}}{x^{pq-qr}} \times \frac{x^{pr}}{x^{qr}}$$

$$= \frac{x^{pq-pr} \times x^{pr}}{x^{pq-qr} \times x^{qr}}$$

$$= \frac{x^{pq-pr+pr}}{x^{pq-qr+qr}}$$

$$= \frac{x^{pq}}{x^{pq}}$$

$\frac{x^{p(q-r)}}{x^{q(p-r)}} \div \left(\frac{x^q}{x^p}\right)^r = \mathbf{1}$ **Ans**

(iv) Find x if $\boldsymbol{log_3(x^3+1) = 2}$

Sol: $log_3(x^3 + 1) = 2$

Writing in exponential form

$x^3 + 1 = 3^2$

$x^3 + 1 = 9$

$x^3 = 9 - 1$

$x^3 = 8$

Taking cube root on both sides

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$\boldsymbol{x = 2}$ **Ans**

(v) If $\boldsymbol{x = 2 + \sqrt{3}}$ **find the values of** $\boldsymbol{x + \frac{1}{x}}$ **and** $\boldsymbol{x - \frac{1}{x}}$**.**

Sol: Given that

$x = 2 + \sqrt{3}$

$\Rightarrow \quad \frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$\frac{1}{x} = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

$\frac{1}{x} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$\frac{1}{x} = \frac{2-\sqrt{3}}{4-3}$

$\frac{1}{x} = \frac{2-\sqrt{3}}{1}$

$\boldsymbol{\frac{1}{x} = 2 - \sqrt{3}}$

Now

$x + \frac{1}{x} = 2 + \sqrt{3} + 2 - \sqrt{3}$

$x + \frac{1}{x} = 2 + 2$

$\boldsymbol{x + \frac{1}{x} = 4}$ **Ans**

Finally

$x - \frac{1}{x} = 2 + \sqrt{3} - (2 - \sqrt{3})$

$x - \frac{1}{x} = 2 + \sqrt{3} - 2 + \sqrt{3}$

$x - \frac{1}{x} = \sqrt{3} + \sqrt{3}$

$\boldsymbol{x - \frac{1}{x} = 2\sqrt{3}}$ **Ans**

(vi) Factorize the expression $p^2 - x^2 + 2x - 1$

Sol: $p^2 - x^2 + 2x - 1 = p^2 - (x^2 - 2x + 1)$

$= p^2 - [(x)^2 - 2(x)(1) + (1)^2]$

$= (p)^2 - (x-1)^2$

$= [p + (x-1)][p - (x-1)]$

$= \boldsymbol{(p + x - 1)(p - x + 1)}$ **Ans**

(vii) Find the HCF of $x^2 + 2x - 8$, $x^2 - 2x - 24$, and $x^2 + 5x + 4$ by factorization.

Sol:

$$x^2 + 2x - 8 = x^2 + 4x - 2x - 8$$
$$= x(x + 4) - 2(x + 4)$$
$$= (x + 4)(x - 2)$$

$$x^2 - 2x - 24 = x^2 - 6x + 4x - 24$$
$$= x(x - 6) + 4(x - 6)$$
$$= (x - 6)(x + 4)$$

$$x^2 + 5x + 4 = x^2 + 4x + x + 4$$
$$= x(x + 4) + 1(x + 4)$$
$$= (x + 4)(x + 1)$$

H.C.F $= \boldsymbol{(x + 4)}$ **Ans**

(viii) Solved the inequality $\frac{5y}{3} - \frac{1}{3}(1 + y) \leq \frac{2}{3}y - \frac{1}{3}(5 - y)$ where $y \in Z$

Sol:

$$\frac{5y}{3} - \frac{1}{3}(1 + y) \leq \frac{2}{3}y - \frac{1}{3}(5 - y)$$

Multiplying each term by 3

$$3 \times \frac{5y}{3} - 3 \times \frac{1}{3}(1 + y) \leq 3 \times \frac{2}{3}y - 3 \times \frac{1}{3}(5 - y)$$

$$5y - (1 + y) \leq 2y - (5 - y)$$

$$5y - 1 - y \leq 2y - 5 + y$$

$$4y - 1 \leq 3y - 5$$

$$4y - 3y \leq -5 + 1$$

$\boldsymbol{y \leq -4}$ **Ans**

(ix) Solved: $\left|\frac{7x-4}{5}\right| = \frac{2}{5}$

Sol:

$$\left|\frac{7x-4}{5}\right| = \frac{2}{5} \qquad \text{----- (i)}$$

The given equation is equivalent to:

$$\frac{7x-4}{5} = \pm\frac{2}{5}$$

i.e. $\frac{7x-4}{5} = \frac{2}{5}$ or $\frac{7x-4}{5} = -\frac{2}{5}$

$7x - 4 = 2$ or $7x - 4 = -2$

$7x = 2 + 4$ or $7x = -2 + 4$

$7x = 6$ or $7x = 2$

$\boldsymbol{x = \frac{6}{7}}$ or $\boldsymbol{x = \frac{2}{7}}$

Check: Put $x = \frac{6}{7}$ in eq (i)

$$\left|\frac{7\left(\frac{6}{7}\right)-4}{5}\right| = \frac{2}{5}$$

$$\left|\frac{6-4}{5}\right| = \frac{2}{5}$$

$$\left|\frac{2}{5}\right| = \frac{2}{5}$$

$\frac{2}{5} = \frac{2}{5}$ **(True)**

Put $x = \frac{2}{7}$ in eq (i)

$$\left|\frac{7\left(\frac{2}{7}\right)-4}{5}\right| = \frac{2}{5}$$

$$\left|\frac{2-4}{5}\right| = \frac{2}{5}$$

$$\left|\frac{-2}{5}\right| = \frac{2}{5}$$

$\frac{2}{5} = \frac{2}{5}$ **(True)**

Hence, solution set = $\left\{\frac{2}{7}, \frac{6}{7}\right\}$ **Ans**

(x) Draw the graph of $4x - 2y + 6 = 0$ by taking at least 4 ordered pairs.

Sol: Given equation is:

$4x - 2y + 6 = 0$

$-2y = -4x - 6$

$-2y = -2(2x + 3)$

$y = 2x + 3$ ---- (i)

Now put $x = 0$ in eq (i)

$y = 2(0) + 3 = 0 + 3 = 3$ (0,3)

Now put $x = 1$ in eq (i)

$y = 2(1) + 3 = 2 + 3 = 5$ (1,5)

Now put $x = -1$ in eq (i)

$y = 2(-1) + 3 = -2 + 3 = 1$ (−1,1)

Now put x=2 in eq (i)

$y = 2(2) + 3 = 4 + 3 = 7$ (2,7)

Please plot the above ordered pairs as point on the graph and join those points to draw a line.

(xi) Using the distance formula, show that points A(1,2), B(2,3), and C(3,4) are collinear.

Sol: We know that distance formula is

$|d| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$

$|BC| = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$

$|AC| = \sqrt{(3-1)^2 + (4-2)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = 2\sqrt{2}$

Since |AB|+|BC| = |AC|

Hence, given points A, B and C are collinear.

(xii) Any point on the bisector of an angle is equidistant from its arms. Prove it.

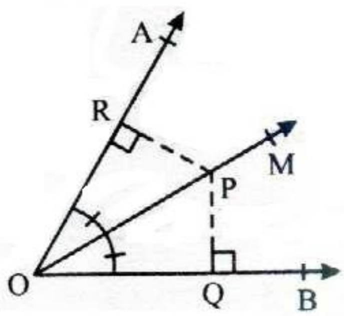
Ans.

Theorem

Any point on the bisector of an angle is equidistant from its arms.

Given

A point P is on $\overrightarrow{OM}$, the bisector of $\angle AOB$.



To Prove

$\overline{PQ} \cong \overline{PR}$ i.e., P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Construction

Draw $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$

Proof

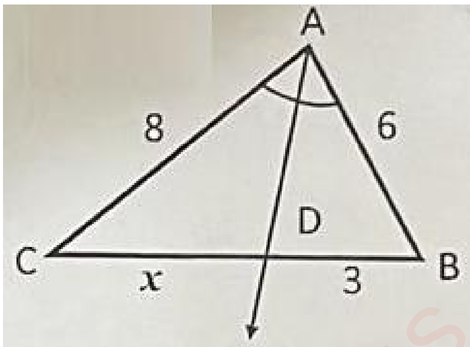
Statements	Reasons
In $\Delta POQ \longleftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	common
$\angle PQO \cong \angle PRO$	construction
$\angle POQ \cong \angle POR$	given
$\therefore$ $\Delta POQ \cong \Delta POR$	S.A.A. ≅ S.A.A.
Hence $\overline{PQ} \cong \overline{PR}$	(corresponding sides of congruent triangles)

(xiii) InΔABC, the internal angle bisector of $\angle A$ meets $\overline{CB}$ at the point D such that $m\overline{BD}:m\overline{DC} = m\overline{AB}:m\overline{AC}$. Find the value of x.

Sol: Given that

$$m\overline{BD}:m\overline{DC} = m\overline{AB}:m\overline{AC}$$

Or $$\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$$



Putting values

$$\frac{3}{x} = \frac{6}{8}$$

24 = 6x

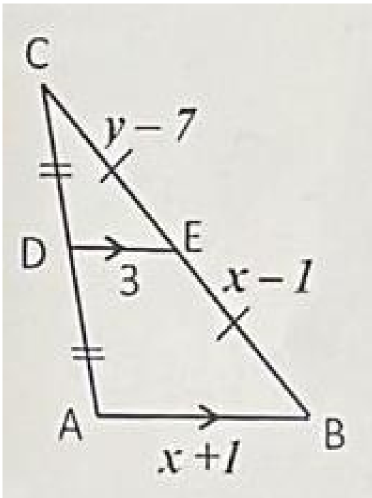
Or 6x = 24

$$x = \frac{24}{6}$$

x = 4

(xiv) In ΔABC, $\overline{DE} \parallel \overline{AB}$ and $\overline{DE} = \frac{1}{2}\ \overline{AB}$. Find the values of x and y.

Sol: Given that



$$\overline{DE} = \frac{1}{2}\ \overline{AB}$$

$$3 = \frac{1}{2}(x + 1)$$

$6 = x + 1$

$6 - 1 = x$

$5 = x$

Or $\boldsymbol{x = 5}$

Also, $\overline{CE} = \overline{BE}$

$y - 7 = x - 1$

$y = x - 1 + 7$

$y = x + 6$

$y = 5 + 6 \qquad (\because x = 5)$

$\boldsymbol{y = 11}$

Section C

Q3. Solve the system of linear equations $2x + 5y = -2,\ 4x + 7y = 2$ by using the matrix inversion method.

Sol: Given equations are:

$2x + 5y = -2$

$4x + 7y = 2$

Writing above equations in matrix form

$$\begin{bmatrix} 2 & 5 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Or $AX = B$ Where $\begin{bmatrix} 2 & 5 \\ 4 & 7 \end{bmatrix} = A$, $\begin{bmatrix} x \\ y \end{bmatrix} = X$ and $\begin{bmatrix} -2 \\ 2 \end{bmatrix} = B$

Or $X = A^{-1}B$ .... (i)

Where $A^{-1} = \dfrac{\text{Adj A}}{|A|}$

Now $|A| = \begin{vmatrix} 2 & 5 \\ 4 & 7 \end{vmatrix} = (2)(7) - (4)(5) = 14 - 20 = -6$

Therefore, $A^{-1} = \dfrac{\begin{bmatrix} 7 & -5 \\ -4 & 2 \end{bmatrix}}{-6}$

Putting values in eq (i)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\begin{bmatrix} 7 & -5 \\ -4 & 2 \end{bmatrix}}{-6} \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-6} \begin{bmatrix} (7)(-2) + (-5)(2) \\ (-4)(-2) + (2)(2) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-6} \begin{bmatrix} -14 - 10 \\ 8 + 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-6} \begin{bmatrix} -24 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-24}{-6} \\ \frac{12}{-6} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

Hence, $\boldsymbol{x = 4}$ and $\boldsymbol{y = -2}$