

Version No.			

ROLL NUMBER						

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 1  1  1  1  
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Answer Sheet No. \_\_\_\_\_

Sign. of Candidate \_\_\_\_\_

Sign. of Invigilator \_\_\_\_\_

### MATHEMATICS HSSC-II (2<sup>nd</sup> Set)

#### SECTION – A (Marks 20)

Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

**Q.1 Fill the relevant bubble for each part. All parts carry one mark.**

- (1) What is the evaluated value of  $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$  ?
- A. 0  B.  $\frac{0}{0}$    
 C.  $\frac{1}{2}$   D. -1
- (2) If  $f(x) = \sqrt{x^2 - 4}$  then Range of  $f$  is:
- A.  $R$   B.  $R^+$    
 C.  $R^-$   D.  $N$
- (3) What evaluates  $\lim_{t \rightarrow \infty} \left(\frac{9t}{1+9t}\right)^{-t}$  ?
- A.  $e^9$   B.  $e^{-9}$    
 C.  $e^{1/9}$   D.  $e^{-1/9}$
- (4) If  $x = \theta + 2$  and  $y = \theta^2 - \theta$ , then  $\frac{dy}{dx}$  is:
- A.  $\frac{1}{2\theta-1}$   B.  $2\theta - 1$    
 C.  $2\theta + 1$   D.  $\frac{1}{2\theta+1}$
- (5) If  $f(x) = e^{-x} + \sin 2x - x^2$ , then  $f'(0)$  is:
- A. -1  B. 0   
 C. 1  D. 2
- (6) If  $\coth y = x$ , then  $\frac{dy}{dx}$  is:
- A.  $-\frac{1}{1-x^2}$   B.  $\frac{1}{1-x^2}$    
 C.  $\frac{1}{1+x^2}$   D.  $-\frac{1}{1+x^2}$
- (7) If  $f(x) = x^2 - x$  when  $x$  changes from 2 to 2.01, then  $dy$  is:
- A. -0.03  B. -0.02   
 C. 0.02  D. 0.03

- (8) If  $\int_{-2}^5 f(x)dx = 12$  and  $\int_2^5 f(x)dx = 5$ , then  $\int_{-2}^2 f(x)dx = ?$
- A. 2  B. 7   
 C. 17  D. -7
- (9) The integral of  $\frac{\sqrt{1+\cot x}}{\sin^2 x}$  w.r.t.  $x$  is:
- A.  $-\frac{2}{3}(1+\cot x)^{\frac{3}{2}} + C$   B.  $\frac{2}{3}(1+\cot x)^{\frac{3}{2}} + C$    
 C.  $-\frac{2}{3}(1+\cot x)^{-\frac{1}{2}} + C$   D.  $\frac{3}{2}(1+\cot x)^{\frac{3}{2}} + C$
- (10) What is the acute angle between the lines  $y = 3x + 2$  and  $y = 4x + 9$  ?
- A.  $4.4^\circ$   B.  $28.3^\circ$    
 C.  $5.2^\circ$   D.  $18.6^\circ$
- (11) What is the perpendicular distance between point  $(2, 1)$  and line  $4x - 2y + 5 = 0$ ?
- A. 1  B. 2   
 C. 3  D. 4
- (12) Which one of the following lines is parallel to  $3x - 2y + 6 = 0$  ?
- A.  $3x + 2y - 12 = 0$   B.  $12x + 18y = 15$    
 C.  $4x - 9y = 6$   D.  $15x - 10y - 9 = 0$
- (13) Which one of the following inequalities is true for the point  $(4, -2)$ ?
- A.  $x + y > 3$   B.  $x < 3y$    
 C.  $x - y < 10$   D.  $2x - 3y < 12$
- (14) In which of the quadrants does the solution region of the inequalities  $x \leq -1$  and  $y > 1$  lie?
- A. I  B. II   
 C. III  D. IV
- (15) The points of intersection of  $y = mx + c$  and  $x^2 + y^2 = a^2$  are Imaginary if:
- A.  $a^2(1 + m^2) > 0$   B.  $a^2(1 + m^2) < c^2$    
 C.  $a^2(1 + m^2) > c^2$   D.  $a^2(1 + m^2) = c^2$
- (16) The lengths of the major and minor axes of an ellipse are 10 m and 8 m, respectively. Find the distance between the foci.
- A. 3  B. 4   
 C. 5  D. 6
- (17) The graph represented by  $x = \cos^2 t$  and  $y = 2\sin t$  is:
- A. Circular  B. Parabolic   
 C. Elliptic  D. Hyperbolic
- (18) If  $0.5\mathbf{i} + 0.8\mathbf{j} + c\mathbf{k}$  is a unit vector, then value of  $c$  is:
- A.  $\sqrt{0.11}$   B. 0.11   
 C.  $\sqrt{0.89}$   D. 0.89
- (19) At what angle between the vectors  $\mathbf{a}$  and  $\mathbf{b}$ ,  $|\mathbf{a} \cdot \mathbf{b}| = |\mathbf{a} \times \mathbf{b}|$ ?
- A. 0  B.  $\frac{\pi}{4}$    
 C.  $\frac{\pi}{2}$   D.  $\pi$
- (20) What results  $\mathbf{i} \cdot (\mathbf{j} \times \mathbf{k}) + \mathbf{j} \cdot (\mathbf{k} \times \mathbf{i}) + \mathbf{k} \cdot (\mathbf{i} \times \mathbf{j})$ ?
- A. -1  B. 0   
 C. 1  D. 3

Federal Board HSSC-II Examination  
Mathematics Model Question Paper  
(Curriculum 2000)

Time allowed: 2.35 hours

Total Marks: 80

Note: Attempt any twelve parts from section 'B' and any four questions from 'C'. Questions therein are to be answered on the separately provided Answer Book. Write your answers neatly and legibly.

**SECTION – B (Marks 48)**

**Q.2** Attempt any **TWELVE** parts. All parts carry equal marks. (12 × 4 = 48)

- i. Evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos 5x}{1 - \cos 7x}$
- ii. A factory's annual sale for  $x$  years is given by  $S(x) = \frac{15x^3 + 23x + 45}{x^2 - 3x + 25}$   
Find the sale for the indicated years
  - a.  $\lim_{x \rightarrow 3} S(x)$
  - b.  $\lim_{x \rightarrow 15} S(x)$
- iii. If  $y = (2x - 3)^{-5}$ , then find  $\frac{dy}{dx}$  by first principle method.
- iv. Find the extreme values of  $f(x) = \sin 2x + 2\cos x$ .
- v. Differentiate  $y = \ln \left[ \tan^{-1} \left( \frac{2x}{1+x^2} \right) \right]$  w.r.t.  $x$ .
- vi. Evaluate  $\int_{-3}^4 [2x + |x + 1|] dx$
- vii. Integrate  $\sin^5 x$  with respect to  $x$ .
- viii. Evaluate  $\int x\sqrt{x+2} dx$ .
- ix. The lines  $y = 2x$ ,  $2x + y - 12 = 0$  and  $y = 2$  enclose a triangular region on  $xy$  - plane. Find
  - a. the coordinates of the vertices of the triangular region.
  - b. the area of the triangular region.
- x. Find the equation of the straight line through origin perpendicular to the line  $3x - 2y + 4 = 0$ . Also find the point of intersection of the two lines?
- xi. Find coordinates of the points of intersection of the line  $3x - y = 16$  and the circle  $3x^2 + 3y^2 - 12x - 15y - 45 = 0$
- xii. Find distance between the directrices of the ellipse  $9x^2 + 13y^2 = 117$
- xiii. Find the equation of a parabola with focus at  $(0, 8)$ , equation of axis of symmetry is  $y = 8$ , distance from focus to directrix is 4 to the left.
- xiv. Determine  $\lambda$  and  $\mu$  by using vectors, such that the points  $A(-1, 3, 2)$ ,  $B(-4, 2, -2)$ ,  $C(5, \lambda, \mu)$  are collinear.
- xv. Using vectors, find the angle  $\hat{ABC}$  if  $A(1, 7, 2)$ ,  $B(3, 3, 4)$ ,  $C(2, 5, 1)$  are the points in space.
- xvi. If  $\underline{a} = 2\underline{i} - \underline{j} + \underline{k}$  and  $\underline{b} = \underline{i} - 3\underline{j} - 5\underline{k}$ , find vectors of magnitude 7 and perpendicular to  $\underline{a}$  and  $\underline{b}$ .

## SECTION – C (Marks 32)

**Note:** Attempt any **FOUR** questions. All questions carry equal marks.

(4 × 8 = 32)

**Q.3** For what values of  $l$  and  $m$ , the function

$$g(x) = \begin{cases} lx + 5 & \text{if } -5 < x < -2 \\ mx^2 - 2 & \text{if } x = -2 \\ x^3 - 5 & \text{if } x > -2 \end{cases} \quad \text{is continuous } \forall x.$$

**Q.4** If  $y = (\sin^{-1} x)^2$ , then show that  $(1 - x^2)y_3 - 3xy_2 - y_1 = 0$ .

**Q.5** The acceleration of a moving particle is  $2t + 4$ . Find

- Velocity of the particle at  $v(0) = 20 \text{ ms}^{-1}$
- Displacement of the particle at  $s(0) = 0$

**Q.6** The line  $5x + 6y = 30$  cuts  $x$  –axis at point  $P$  and  $y$  –axis at point  $Q$ . Find

- the coordinates of points  $P$  and  $Q$ .
- the length of  $PQ$ .
- the point  $R$  on  $PQ$  such that  $R$  is equidistant from  $x$  –axis and  $y$  –axis.
- the equation of the line  $OR$  where  $O$  is the origin.

**Q.7** Saira wants to buy bananas and apples at Rs.6 and Rs.10 each, respectively. She must buy at least one of each fruit but the capacity of her basket is not more than 5 fruits. Shopkeeper's profit on each banana is Rs.26 and on each apple it is Rs. 10.

- Write down the three inequalities.
- Draw graphs on the same axis to show these conditions.
- Shade the area containing the solution set.
- Determine how many of each fruit Saira must buy for the shopkeeper to get the maximum profit.

**Q.8** Find the equation of hyperbola with centre at origin, conjugate axis along  $x$ -axis, eccentricity  $\sqrt{7}$  and sum of lengths of whose axes is 32. Also find equations of the asymptotes of the hyperbola.

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**MATHEMATICS HSSC-II (2<sup>nd</sup> Set)**  
**Student Learning Outcomes Alignment Chart**  
 National Curriculum 2000

Sec-A	Q 1	Contents and Scope	Student Learning Outcomes
	i	Limits of Important Functions	(a) Limit of the following functions at $x = a$ $\frac{x^n - a^n}{x - a}$ , $\frac{x - a}{\sqrt{x} - \sqrt{a}}$
	ii	Revision of the work done in the previous classes	Function, its domain and range
	iii	Limits of Functions and Theorems on Limits	Intuitive notion of limit of function at a point and at $\alpha$ , illustration with suitable examples.
	iv	The Chain Rule	Explanation and application of chain rule for composite functions and functions defined by parametric functions.
	v	Differentiation of Functions other than algebraic	To find the derivatives of trigonometric, exponential functions using chain and other rules.
	vi	Differentiation of Functions other than algebraic	To find the derivatives of inverse hyperbolic functions using chain and other rules.
	vii	Differentials	To have the concept of differentials and to a) distinguish between $dy$ and $\delta y$
	viii	Definite Integrals	c) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
	ix	Theorems on Anti-derivatives	To know the theorems (without proof) on anti-derivatives of b) sum and difference of functions and their applications
	x	Two and Three Straight lines	Be able to find: c) Acute angle between two straight lines, condition of their parallelism and perpendicularity.
	xi	Equations of Straight lines	To know the position of a point with respect to a line and to find the distance of a point from a line.
	xii	Two or Three Straight lines	Be able to find: a) concept of the slope of a line c) acute angle between two straight lines, condition of their parallelism and perpendicularity.
	xiii	Linear Inequalities and their Graphs	To know the meanings of linear inequalities in two variables and their solutions be graphically illustrated.
	xiv	Linear Inequalities and their Graphs	Determine graphically the region bounded by 2 or 3 simultaneous inequalities of non-negative variables.
	xv	Tangents and Normals	To find: a) the points of intersection of a circle with a line including the condition of tangency.
	xvi	Ellipse and its Elements	To know the concept of an ellipse and its elements

	xvii	Parabola and its elements	To know the concept of a parabola and its elements
	xviii	Introduction of a Vector in Space	Fundamental unit vectors $\underline{i}$ , $\underline{j}$ , $\underline{k}$ Components of a vector $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$ : Magnitude of a vector.
	xix	Scalar Product of two Vectors Vector Product of two Vectors	To know the definition of scalar (dot) product of two vectors i.e. $\underline{a} \cdot \underline{b} = ab \cos \theta$ To know the definition of vector (cross) product of two vectors i.e. $\underline{a} \times \underline{b} = ab \sin \theta \hat{n}$ where $\theta$ is the measure of the angle between vectors $\underline{a}$ and $\underline{b}$ . angle between two vectors
	xx	Scalar Triple Product of Vectors	To know the definition of the Scalar triple product of vectors. $(\underline{a} \times \underline{b}) \cdot \underline{c}$ , $(\underline{b} \times \underline{c}) \cdot \underline{a}$ , $(\underline{c} \times \underline{a}) \cdot \underline{b}$
<b>Sec-B</b>	<b>Q 2</b>		
	i	Limits of Important Functions	Limit of the function at $x = 0$ and their application in evaluation of the limits of the trigonometric functions
	ii	Limits of Functions and Theorems on Limits	Theorems on sum, difference, product and quotient of function
	iii	Differentiation of Algebraic Expressions	To be able to calculate the derivatives of $y = \frac{1}{(ax+b)^n}$ , $n = 1, 2, 3, \dots$ By definition (ab-initio)
	iv	Extreme Values	To have the concept of maximum and minimum values and critical points of a function.
	v	Differentiation of Functions other than algebraic	To find the Derivative of Inverse Trigonometric and Logarithmic Functions using Chain and other rules.
	vi	Definite Integrals	c. $\int_a^b f(x)dx = -\int_b^a f(x)dx$ $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
	vii	Integration by Substitution	To know and be able to integrate by applying the method of substitution in the integration of functions.
	viii	Integration by Parts	To know and be able to find the anti-derivatives of functions by parts.
	ix	Two and Three Straight lines	The point of intersection of two straight lines. To find the area of a triangle whose vertices are given.
	x	Equations of straight lines.	To transform the linear equation $ax + by + c = 0$ in the standard forms listed (point-slope form). The point of intersection of two straight lines.
	xi	Tangents and Normals	To find the point of intersection of a circle with a line.
	xii	Ellipse and its Elements	To know the concept of Ellipse and its elements (centre, foci, eccentricity, vertices, major and minor axes, focal chords, latera

			recta and directrices). $(x - h)^2 + (y - k)^2 = r^2$
	xiii	Equation of a Parabola with given elements	To find the equation of a parabola with the following given elements. • focus and directrix
	xiv	Introduction of Vector in space	Parallel, Collinear and Coplanar vectors.
	xv	Scalar Product of two Vectors	Angle between two vectors
	xvi	Vector Product of two Vectors	To know the determinantal expression of the vector product of two vectors. Magnitude of a vector, Unit vector
<b>Sec-C</b>	<b>Q No</b>		
	3	Continuous and Discontinuous Functions	To understand the concept of continuity of a function at a point and in an interval intuitively.
	4	Successive Differentiation	• To find 2 <sup>nd</sup> , 3 <sup>rd</sup> and 4 <sup>th</sup> derivatives of algebraic, trigonometric, exponential and logarithmic functions.
	5	Differential Equations	To have the concept of differential equations and its order. To be able to solve differential equations of first order with variables separable; concept of initial conditions and simple applications.
	6	Coordinate System Equations of Straight Lines	Be able to: b) derive the distance formula. c) find the midpoint of the line segment. d) Derivation of the standard forms of the equations of the straight lines; two points form; intercepts form.
	7	Linear Programming	To have the concepts of simple linear programming and of optimal solution of the linear objective functions and to find the optimal solution of the linear objective functions by graphical methods.
	8	Equation of Hyperbola with given elements	To find the equation of a hyperbola with the following elements: Transverse and conjugate axis with centre at origin. Eccentricity, latera recta and transverse axis.

## MATHEMATICS HSSC-II (2<sup>nd</sup> Set)

### Table of Specifications

Topics	1. Functions and Limits	2. Differentiation	3. Integration	4. Introduction to Analytic Geometry	5. Linear Inequalities and Programming	6. Conic Section	7. Vectors	Total marks	% age
Knowledge based	1ii(1) 2i(4) 3(8)	1vi(1) 2iv(4)	1vii(1) 2viii(4)	1xi(1) 2ix(2) 2x(2)	1xiii(1)	2xi(4)	1xviii(1) 1xx(1) 2xvi(4)	39	29.54%
Comprehension based	1i(1)	1v(1) 2iii(4) 2v(4) 4(8)	1ix(1) 2vii(4)	1x(1) 1xii(1) 2x(2) 6(8)	1xiv(1)	1xv(1) 1xvi(1) 2xii(4) 1xvii(1) 8(8)	1xix(1) 2xv(4) 2xiv(4)	60	45.45%
Application based	2ii(4) 1iii(1)	1iv(1)	2vi(4) 1viii(1) 5(8)	2ix(2)	7(8)	2xiii(4)		33	25%
Total marks for each topic	19	23	23	19	10	23	15	132	100%

**KEY:**

1(1)(01)

Question No (Part No.) (Allocated Marks)

**Note:** (i) The policy of FBISE for knowledge based questions, understanding based questions and application based questions is approximately as follows:

- a) 30% knowledge based.
- b) 50% understanding based.
- c) 20% application based.

(ii) The total marks specified for each unit/content in the table of specification is only related to this model question paper.

(iii) The level of difficulty of the paper is approximately as follows:

- a) 40% easy
- b) 40% moderate
- c) 20% difficult