Version No.			
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

ROLL NUMBER						
0	0	0	0	0	0	0
1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9

Answer Sheet No.____________

Sign. of Candidate ____________

Sign. of Invigilator ____________

MATHEMATICS HSSC–II (2nd Set)

SECTION – A (Marks 20)

Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

Q.1 Fill the relevant bubble for each part. All parts carry one mark.

(1) What is the evaluated value of $\lim_{x \to 1} \frac{\sqrt{x}-1}{x-1}$?

A. 0 B. $\frac{0}{0}$

C. $\frac{1}{2}$ D. -1

(2) If $f(x) = \sqrt{x^2 - 4}$ then Range of f is:

A. R B. R^+

C. R^- D. N

(3) What evaluates $\lim_{t \to \infty} \left(\frac{9t}{1+9t}\right)^{-t}$?

A. e^9 B. e^{-9}

C. $e^{1/9}$ D. $e^{-1/9}$

(4) If $x = \theta + 2$ and $y = \theta^2 - \theta$, then $\frac{dy}{dx}$ is:

A. $\frac{1}{2\theta-1}$ B. $2\theta - 1$

C. $2\theta + 1$ D. $\frac{1}{2\theta+1}$

(5) If $f(x) = e^{-x} + sin2x - x^2$, then $f'(0)$ is:

A. -1 B. 0

C. 1 D. 2

(6) If $cothy = x$, then $\frac{dy}{dx}$ is:

A. $-\frac{1}{1-x^2}$ B. $\frac{1}{1-x^2}$

C. $\frac{1}{1+x^2}$ D. $-\frac{1}{1+x^2}$

(7) If $f(x) = x^2 - x$ when x changes from 2 to 2.01, then dy is:

A. -0.03 B. -0.02

C. 0.02 D. 0.03

(8) If $\int_{-2}^{5} f(x)dx = 12$ and $\int_{2}^{5} f(x)dx = 5$, then $\int_{-2}^{2} f(x)dx = ?$

A. 2 ◯ B. 7 ◯
C. 17 ◯ D. -7 ◯

(9) The integral of $\frac{\sqrt{1+cotx}}{\sin^2 x}$ w.r.t. x is:

A. $-\frac{2}{3}(1+cotx)^{\frac{3}{2}}+C$ ◯ B. $\frac{2}{3}(1+cotx)^{\frac{3}{2}}+C$ ◯
C. $-\frac{2}{3}(1+cotx)^{-\frac{1}{2}}+C$ ◯ D. $\frac{3}{2}(1+cotx)^{\frac{3}{2}}+C$ ◯

(10) What is the acute angle between the lines $y = 3x + 2$ and $y = 4x + 9$?

A. 4.4° ◯ B. 28.3° ◯
C. 5.2° ◯ D. 18.6° ◯

(11) What is the perpendicular distance between point $(2, 1)$ and line $4x - 2y + 5 = 0$?

A. 1 ◯ B. 2 ◯
C. 3 ◯ D. 4 ◯

(12) Which one of the following lines is parallel to $3x - 2y + 6 = 0$?

A. $3x + 2y - 12 = 0$ ◯ B. $12x + 18y = 15$ ◯
C. $4x - 9y = 6$ ◯ D. $15x - 10y - 9 = 0$ ◯

(13) Which one of the following inequalities is true for the point $(4, -2)$?

A. $x + y > 3$ ◯ B. $x < 3y$ ◯
C. $x - y < 10$ ◯ D. $2x - 3y < 12$ ◯

(14) In which of the quadrants does the solution region of the inequalities $x \leq -1$ and $y > 1$ lie?

A. I ◯ B. II ◯
C. III ◯ D. IV ◯

(15) The points of intersection of $y = mx + c$ and $x^2 + y^2 = a^2$ are Imaginary if:

A. $a^2(1 + m^2) > 0$ ◯ B. $a^2(1 + m^2) < c^2$ ◯
C. $a^2(1 + m^2) > c^2$ ◯ D. $a^2(1 + m^2) = c^2$ ◯

(16) The lengths of the major and minor axes of an ellipse are 10 m and 8 m, respectively. Find the distance between the foci.

A. 3 ◯ B. 4 ◯
C. 5 ◯ D. 6 ◯

(17) The graph represented by $x = \cos^2 t$ and $y = 2sint$ is:

A. Circular ◯ B. Parabolic ◯
C. Elliptic ◯ D. Hyperbolic ◯

(18) If $0.5\underline{i} + 0.8\underline{j} + c\underline{k}$ is a unit vector, then value of c is:

A. $\sqrt{0.11}$ ◯ B. 0.11 ◯
C. $\sqrt{0.89}$ ◯ D. 0.89 ◯

(19) At what angle between the vectors $\underline{a}$ and $\underline{b}$, $|\underline{a}.\underline{b}| = |\underline{a} \times \underline{b}|$?

A. 0 ◯ B. $\frac{\pi}{4}$ ◯
C. $\frac{\pi}{2}$ ◯ D. π ◯

(20) What results $\underline{i}.(\underline{j} \times \underline{k}) + \underline{j}.(\underline{k} \times \underline{i}) + \underline{k}.(\underline{i} \times \underline{j})$?

A. -1 ◯ B. 0 ◯
C. 1 ◯ D. 3 ◯

Model Question Paper HSSC-II

Mathematics

(2^{nd} Set)SOLUTION

Section – A

Q1.

1	C	2	B	3	C	4	B	5	A	6	B	7	D	8	B	9	A	10	A
11	B	12	D	13	C	14	B	15	B	16	D	17	B	18	A	19	B	20	D

Section – B

Q2.

(i) $\lim\limits_{x\to 0}\left(\frac{1-cos5x}{1-cos7x}\right)$

$$= \lim_{x\to 0}\frac{2\sin^2\left(\frac{5x}{2}\right)}{2\sin^2\left(\frac{7x}{2}\right)} \qquad = \lim_{x\to 0}\left(\frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{7x}{2}\right)}\right)^2$$

$$= \lim_{x\to 0}\left(\frac{\sin\left(\frac{5x}{2}\right)}{5x/2}\times\frac{7x/2}{\sin\left(\frac{7x}{2}\right)}\times\frac{5x/2}{7x/2}\right)^2$$

$$= \frac{25}{49}\times\left[\lim_{x\to 0}\left(\frac{\sin\left(\frac{5x}{2}\right)}{\frac{5x}{2}}\right)\right]^2\times\left[\lim_{x\to 0}\left(\frac{7x/2}{\sin\left(\frac{7x}{2}\right)}\right)\right]^2$$

$$= \frac{25}{49}\times(1)^2\times(1)^2 = \frac{25}{49} \qquad \because \lim_{x\to 0}\frac{sinx}{x} = 1$$

(ii) $S(x) = \frac{15x^3+23x+45}{x^2-3x+25}$

(a) $\lim\limits_{x\to 3}S(x) = ?$

$$\lim_{x\to 3}S(x) = \lim_{x\to 3}\left(\frac{15x^3+23x+45}{x^2-3x+25}\right) = \frac{15(3)^3+23(3)+45}{3^2-3(3)+25} = 20.76$$

(b) $\lim\limits_{x\to 15}S(x) = ?$

$$\lim_{x\to 15}S(x) = \lim_{x\to 15}\left(\frac{15x^3+23x+45}{x^2-3x+25}\right) = \frac{15(15)^3+23(15)+45}{(15)^2-3(15)+25}\lim_{x\to 15}S(x) = 248.85$$

(iii) $y = (2x-3)^{-5}$

Taking increment to both sides

$y+\delta y = [2(x+\delta x)-3]^{-5}$

Subtracting above equations as

$$y + \delta y - y = [2(x + \delta x) - 3]^{-5} - (2x - 3)^{-5}$$
$$\delta y = [2x + 2\delta x - 3]^{-5} - (2x - 3)^{-5}$$
$$\delta y = [(2x - 3) + 2\delta x]^{-5} - (2x - 3)^{-5}$$
$$\delta y = (2x - 3)^{-5}\left[1 + \frac{2\delta x}{2x - 3}\right]^{-5} - (2x - 3)^{-5}$$
$$\delta y = (2x - 3)^{-5}\left[\left(1 + \frac{2\delta x}{2x - 3}\right)^{-5} - 1\right]$$
$$\delta y = (2x - 3)^{-5}\left[\left\{1 + (-5)\left(\frac{2\delta x}{2x - 3}\right) + \frac{(-5)(-6)}{2!}\left(\frac{2\delta x}{2x - 3}\right)^2 + \cdots\right\} - 1\right]$$
$$\delta y = (2x - 3)^{-5}\left[(-5)\left(\frac{2\delta x}{2x - 3}\right) + \frac{(-5)(-6)}{2!}\left(\frac{2\delta x}{2x - 3}\right)^2 + \cdots\right]$$
$$\delta y = (2x - 3)^{-5}(-5)\left(\frac{2\delta x}{2x - 3}\right)\left[1 + \frac{(-6)}{2!}\left(\frac{2\delta x}{2x - 3}\right)^1 + \cdots\right]$$
$$\frac{\delta y}{\delta x} = -10(2x - 3)^{-6}\left[1 + \frac{(-6)}{2!}\left(\frac{2\delta x}{2x - 3}\right)^1 + \cdots\right]$$
$$\lim_{\delta x \to 0}\frac{\delta y}{\delta x} = -10(2x - 3)^{-6}\lim_{\delta x \to 0}\left[1 + \frac{(-6)}{2!}\left(\frac{2\delta x}{2x - 3}\right)^1 + \cdots\right]$$
$$\frac{dy}{dx} = -10(2x - 3)^{-6}[1 + 0 + \cdots]$$
$$\frac{dy}{dx} = -10(2x - 3)^{-6}$$

(iv) $f(x) = sin2x + 2cosx$

Differentiating w.r.t.x

$f'(x) = 2cos2x - 2sinx$; $f''(x) = -4sin2x - 2cosx$

For extreme values put $f'(x) = 0$

$2cos2x - 2sinx = 0$

$cos2x - sinx = 0$

$1 - 2\sin^2 x - sinx = 0$

$2\sin^2 x + sinx - 1 = 0$

$(sinx + 1)(2sinx - 1) = 0$

$sinx + 1 = 0$ or $2sinx - 1 = 0$

$sinx = 1$ or $sinx = \frac{1}{2}$

$x = \frac{\pi}{2}$ or $x = \frac{\pi}{6}$

$$f''\left(\frac{\pi}{2}\right) = -4sin\pi - 2cos\frac{\pi}{2} = 0$$
$$f''\left(\frac{\pi}{6}\right) = -4sin\left(\frac{\pi}{3}\right) - 2cos\left(\frac{\pi}{6}\right) = -3\sqrt{3} < 0$$

f shows its maximum value at $x = \frac{\pi}{6}$

$$f_{max}:\ f\left(\frac{\pi}{6}\right) = sin\left(\frac{\pi}{3}\right) + 2cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + \frac{2\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$$

(v) $y = ln\left[\tan^{-1}\left(\frac{2x}{1+x^2}\right)\right]$

Differentiating w.r.t.x

$$\frac{dy}{dx} = \left[\tan^{-1}\left(\frac{2x}{1 + x^2}\right)\right]^{-1}\frac{d}{dx}\left[\tan^{-1}\left(\frac{2x}{1 + x^2}\right)\right]$$

$$\frac{dy}{dx}=\left[\tan^{-1}\left(\frac{2x}{1+x^2}\right)\right]^{-1}\frac{1}{1+\left(\frac{2x}{1+x^2}\right)^2}\frac{d}{dx}\left[\left(\frac{2x}{1+x^2}\right)\right]$$

$$\frac{dy}{dx}=\left[\tan^{-1}\left(\frac{2x}{1+x^2}\right)\right]^{-1}\frac{(1+x^2)^2}{(1+x^2)^2+4x^2}\left[\frac{(1+x^2)(2)-2x(2x)}{(1+x^2)^2}\right]$$

$$\frac{dy}{dx}=\left[\tan^{-1}\left(\frac{2x}{1+x^2}\right)\right]^{-1}\frac{2-2x^2}{(1+x^2)^2+4x^2}$$

(vi) $\int_{-3}^{4}[2x+|x+1|]dx$

$=\int_{-3}^{-1}[2x-(x+1)]dx+\int_{-1}^{4}[2x+(x+1)]dx$

$=\int_{-3}^{-1}[x-1]dx+\int_{-1}^{4}[3x+1]dx$

$$=\left[\frac{x^2}{2}-x\right]_{-3}^{-1}+\left[\frac{3x^2}{2}+x\right]_{-1}^{4}$$

$$=\left[\left(\frac{1}{2}+1\right)-\left(\frac{9}{2}+3\right)\right]+\left[\left(\frac{48}{2}+4\right)-\left(\frac{3}{2}-1\right)\right]$$

$$=\left[\frac{3}{2}-\frac{15}{2}\right]+\left[\frac{56}{2}-\frac{1}{2}\right]=\frac{43}{2}$$

(vii) $I=\int\sin^5 x\,dx$

$$I=\int(sinx)(sinx)^4dx=\int(sinx)(\sin^2 x)^2dx=\int(1-\cos^2 x)^2(sinx)dx$$

Let $y=cosx \;\Rightarrow dy=-sinxdx \Rightarrow -dy=sinxdx$

$$I=-\int(1-y^2)^2dy=-\int(y^4-2y^2+1)dy$$

$$I=-\frac{y^5}{5}+\frac{2y^3}{3}-y+C$$

$$I=-\frac{y^5}{5}+\frac{2y^3}{3}-y+C$$

$$I=-\frac{\cos^5 y}{5}+\frac{2\cos^3 y}{3}-cosy+C$$

(viii) $I=\int x\sqrt{x+2}\,dx$

Let $t=x+2 \Rightarrow x=t-2 \;\Rightarrow dx=dt$

$$I=\int(t-2)\sqrt{t}\,dt$$

$$I=\int\left(t^{3/2}-2t^{1/2}\right)dt$$

$$I=\frac{t^{5/2}}{5/2}-\frac{2t^{3/2}}{3/2}+C$$

$$I=\frac{2}{5}(x+2)^{5/2}-\frac{4}{3}(x+2)^{3/2}+C$$

(ix) (a) $l_1:\ y=2x$ $\quad l_2:\ 2x+y-12=0$ $\quad l_3:\ y=2$

Solving $l_1\&l_2$

Substitute l_1 in l_2

$2x+2x-12 \Longrightarrow 4x=12 \Longrightarrow x=3$

Now substitute $x=3$ in l_1 , $y=2(3)=6$

l_1 & l_2 intersect at A(3,6)

Solving $l_2 \& l_3$

Substitute l_3 in l_2 we get

$2x + 2 - 12 = 0 \Longrightarrow 2x - 10 = 0 \;\Rightarrow x = 5$

Substitute $x = 5$ in $\boldsymbol{l_1}$, $y = 2(5) = 10$

l_2 & l_3 intersect at $\mathrm{B}(5,10)$

Solving $l_1 \& l_3$

Substitute l_3 in l_1 $\quad 2 = 2x \Longrightarrow x = 1$

l_1 & l_3 intersect at $\mathrm{C}(1,2)$

Hence $\mathrm{A}(3,6), \mathrm{B}(5,10), \mathrm{C}(1,2)$ are the vertices of the triangular region.

(b) Area of the triangular region having vertices

$(x_1, y_1) = \mathrm{A}(3,6),\; (x_2, y_2) = \mathrm{B}(5,10),\; (x_3, y_3) = \mathrm{C}(1,2)$ is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & 6 & 1 \\ 5 & 10 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\Delta = \frac{1}{2}[3(10-2) - 6(5-1) + 1(10-10)]$$

$$\Delta = \frac{1}{2}[24 - 24 + 0] = 0$$

(x) Let l_1: $3x - 2y + 4 = 0$ be the given line.

Slope of l_1: $m_1 = -\frac{3}{-2} = \frac{3}{2}$

Let l_2 be the line perpendicular to l_1

Slope of l_2: $m_2 = \frac{-1}{m_1} = \frac{-1}{\frac{3}{2}} = -\frac{2}{3}$

Equation of l_2 with $m_2 = -\frac{2}{3}$ and through $(x_1, y_1) = (0, 0)$ is

$y - y_1 = m_2(x - x_1)$

$y - 0 = -\frac{2}{3}(x - 0)$

$3y = -2x$

l_2: $2x + 3y = 0$

Finding point of intersection of $l_1,\ l_2$ by Cross Multiplication method

l_1: $3x - 2y + 4 = 0$

l_2: $2x - 3y + 0 = 0$

$\frac{x}{0+12} = \frac{-y}{0-8} = \frac{1}{-9+4}$

$\frac{x}{12} = \frac{y}{8} = \frac{1}{-5}$

$x = \frac{-12}{5}$ and $y = -\frac{8}{5}$

Hence $\left(\frac{-12}{5}, -\frac{8}{5}\right)$ is the point of intersection of $l_1,\ l_2$

(xi) Line: $3x - y = 16$ or $y = 3x - 16$ eqn-1

Circle: $3x^2 + 3y^2 - 12x - 15y - 45 = 0$ eqn-2

Substituting value of y form eqn-1 in eqn-2 to obtain

$3x^2 + 3(3x - 16)^2 - 12x - 15(3x - 16) - 45 = 0$

$3x^2 + 27x^2 - 288x + 768 - 12x - 45x + 240 - 45 = 0$

$30x^2 - 345x + 963 = 0$

$10x^2 - 115x + 321 = 0$

By using the Quadratic formula

$$x = \frac{115 \pm \sqrt{(-115)^2 - 4(10)(321)}}{2(10)} = \frac{115 \pm \sqrt{13225 - 12840}}{20} = \frac{115 \pm \sqrt{385}}{20}$$

$x = 4.77 \text{ or } 6.73$

Now, we substitute these values of x in eqn-2, we get

$y = 3(4.77) - 16$; $y = 3(6.73) - 16$

$y = -1.69$; $y = 4.19$

Hence $(4.77, -1.69)$ and $(6.73, 4.19)$ are the points of intersection of the given line and circle.

(xii) Equation of ellipse: $9x^2 + 13y^2 = 117$

Dividing by 117 $\frac{x^2}{13} + \frac{y^2}{9} = 1$

Compare with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

we get $a^2 = 13$ and $b^2 = 9$

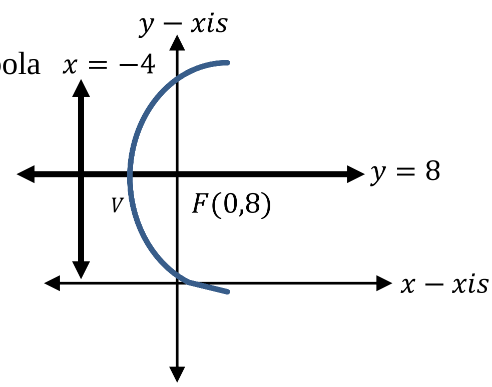
for eccentricity $b^2 = a^2(1 - e^2)$

$9 = 13(1 - e^2)$

$e = \frac{2}{\sqrt{13}}$

Distance between the directrices $= \frac{2a}{e} = \frac{2\sqrt{13}}{\frac{2}{\sqrt{13}}} = 13$

(xiii) Let p: $(y - k)^2 = 4a(x - h)$ be the required parabola



with vertex $V(h, k)$, axis of symmetry $y = 8$

focus $F(0,8)$ and directrix $x = -4$

Since vertex lies in the mid of directrix and focus on the axis of symmetry.

Thus $V(h, k) = V(-2,8) \Rightarrow h = -2, k = 8$

Also $a = |VF| = \sqrt{(-2-0)^2 + (8-8)^2} = 2$

Substituting values in eqn-p

$(y-8)^2 = 4(2)(x+2)$

Hence $(y-8)^2 = 8(x+2)$ is the required equation of Parabola.

(xiv) $\mathrm{A}(-1,3,2),\ \mathrm{B}(-4,\ 2,-2),\ \mathrm{C}(5,\ \lambda,\ \mu)$

If $O(0,0,0)$ be the reference point, then

$p.v\ of\ A = \overrightarrow{OA} = [-1,3,2]$

$p.v\ of\ B = \overrightarrow{\mathrm{OB}} = [-4,2,-2]$

$p.v\ of\ C = \overrightarrow{\mathrm{OC}} = [5,\ \lambda,\ \mu]$

Taking $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = [-4,2,-2] - [-1,3,2] = [-3,-1,-4]$

$\overrightarrow{\mathrm{BC}} = \overrightarrow{\mathrm{OC}} - \overrightarrow{\mathrm{OB}} = [5,\ \lambda,\mu] - [-4,\ 2,-2] = [9,\ \lambda-2,\ \mu+2]$

If A, B and C are collinear, then $\overrightarrow{\mathrm{AB}} = \mathrm{t}\,\overrightarrow{\mathrm{BC}}$ where t is scalar

$[-3,-1,-4] = \mathrm{t}[9,\ \lambda-2,\ \mu+2]$

$[-3,-1,-4] = [9\mathrm{t},(\lambda-2)\mathrm{t},(\mu+2)\mathrm{t}]$

$-3 = 9\mathrm{t} \Rightarrow \mathrm{t} = -\frac{1}{3}$

$-1 = t(\lambda-2) \quad \Rightarrow -1 = -\frac{1}{3}(\lambda-2) \quad \Rightarrow \lambda = 5 \qquad \because \mathrm{t} = -\frac{1}{3}$

$-4 = t(\mu+2) \quad \Rightarrow -4 = -\frac{1}{3}(\mu+2) \quad \Rightarrow \mu = 10 \qquad \because \mathrm{t} = -\frac{1}{3}$

(xv) $A(1,7,2), B(3,3,4), C(2,5,1)$

If $O(0,0,0)$ be the reference point, then

$p.v\ of\ A = \overrightarrow{OA} = [1,7,2]$

$p.v\ of\ B = \overrightarrow{\mathrm{OB}} = [3,3,4]$

$p.v\ of\ C = \overrightarrow{\mathrm{OC}} = [2,5,1]$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = [3,3,4] - [1,7,2] = [2,-4,2]$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = [2,5,1] - [3,3,4] = [-1,2,-3]$

$\overrightarrow{AB}.\overrightarrow{BC} = [2,-4,2].[-1,2,-3] = 2(-1) - 4(2) + 2(-3) = -12$

$|\overrightarrow{AB}| = \sqrt{(2)^2 + (-4)^2 + (2)^2} = \sqrt{24}$

$|\overrightarrow{BC}| = \sqrt{(-1)^2 + (2)^2 + (-3)^2} = \sqrt{14}$

If θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{BC}$, then

$\cos\theta = \frac{\overrightarrow{AB}.\overrightarrow{BC}}{|\overrightarrow{AB}|\,|\overrightarrow{BC}|} = \frac{-12}{\sqrt{24}\sqrt{14}}$

$\theta = 130.54$

(xvi) $\underline{a} = [2,-1,1]\ ,\underline{b} = [1,-3,-5]$

$$\underline{a}\times\underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -1 & 1 \\ 1 & -3 & -5 \end{vmatrix} = 8\underline{i} - 11\underline{j} - 5\underline{k}$$

$|\underline{a}\times\underline{b}| = \sqrt{64+121+25} = \sqrt{210}$

Unit vector $= \frac{\underline{a}\times\underline{b}}{|\underline{a}\times\underline{b}|} = \frac{8\underline{i}-11\underline{j}-5\underline{k}}{\sqrt{210}}$

Required vector $= 7\left(\frac{8\underline{i}-11\underline{j}-5\underline{k}}{\sqrt{210}}\right) = \frac{56}{\sqrt{210}}\underline{i} - \frac{77}{\sqrt{210}}\underline{j} - \frac{35}{\sqrt{210}}$

SECTION C

Q3. $g(x) = \begin{cases} lx+5 & if\ -5 < x < -2 \\ mx^2-2 & if\ x=-2 \\ x^3-5 & if\ x > -2 \end{cases}$ is continuous $\forall\ x$.

Value of the Function

at $x=-2$, $g(x) = mx^2-2$

$$g(-2) = m(-2)^2 - 2 \ = 4m-2$$

Limit of the Function

L.H.Limit: $\lim_{x\to-2^-} g(x) = \lim_{x\to-2^-}(lx+5) = -2l+5$

R.H.Limit: $\lim_{x\to-2^+} g(x) = \lim_{x\to-2^+}(x^3-5) = (-2)^3-5 = -13$

Since $g(x)$ is continuous$\forall\ x$, so $\lim_{x\to-2} g(x)$ exists.

$\lim_{x\to-2^-} g(x) = \lim_{x\to-2^+} g(x)$

$-2l+5 = -13 \quad \Rightarrow\ l = 9$

Value = Limit

$g(-2) = \lim_{x\to-2} g(x)$

$4m-2 = -13$

$m = -\frac{11}{4}$

Q4. $y = (\sin^{-1} x)^2$

Differentiating w.r.t.x

$$y_1 = 2(\sin^{-1} x)\frac{d}{dx}(\sin^{-1} x)$$

$$y_1 = 2\sin^{-1} x \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}y_1 = 2\sin^{-1} x$$

Squaring both sides

$$(1-x^2){y_1}^2 = 4(\sin^{-1} x)^2$$

$$(1-x^2){y_1}^2 = 4y \qquad \because y = (\sin^{-1} x)^2$$

Differentiating w.r.t.x

$$(1-x^2)2y_1y_2 - 2x{y_1}^2 = 4y_1$$

$$(1-x^2)2y_1y_2 - 2x{y_1}^2 - 4y_1 = 0$$

$$2y_1[(1-x^2)y_2 - xy_1 - 2] = 0$$

$$y_1[(1-x^2)y_2 - xy_1 - 2] = 0$$

$(1-x^2)y_2 - xy_1 - 2$ where as $y_1 \neq 0$

Differentiating w.r.t.x

$$(1-x^2)y_3 - 2xy_2 - xy_2 - y_1 = 0$$

$$(1-x^2)y_3 - 3xy_2 - y_1 = 0$$

Q5. Let $a = 2t + 4$

$$\frac{dv}{dt} = 2t + 4$$

$$\int dv = \int (2t+4)dt$$

$$v = 2\left(\frac{t^2}{2}\right) + 4t + C_1$$

$$v = t^2 + 4t + C_1 \longrightarrow Eqn(i)$$

Initially $v(0) = 20ms^{-1}$

$$20 = 0 + 0 + C_1 \ \Rightarrow C_1 = 20$$

$$Eqn(i) \longrightarrow v = (t^2 + 4t + 20)ms^{-1}$$

$$\frac{ds}{dt} = t^2 + 4t + 20$$

$\int ds = \int (t^2 + 4t + 20)\, dt$

$s = \frac{t^3}{3} + 4\left(\frac{t^2}{2}\right) + 20t + C_2$

$s = \frac{t^3}{3} + 2t^2 + 20t + C_2 \;\longrightarrow Eqn(ii)$

Initially $s(0) = 0$

$0 = 0 + 0 + 0 + C_2 \quad \Rightarrow C_2 = 0$

$Eqn(ii) \longrightarrow s = \left(\frac{t^3}{3} + 2t^2 + 20t\right) m$

Q6 Let $l_1 : 5x + 6y = 30$ be the given line

(a) l_1 cuts $x - axis$ at $P(x, 0)$ giving $5x + 6(0) = 30 \Rightarrow x = 6 \Rightarrow P(6,0)$

l_1 cuts $y - axis$ at $Q(0, y)$ giving $5(0)x + 6y = 30 \Rightarrow y = 5 \Rightarrow Q(0,5)$

(b) $|\overline{PQ}| = \sqrt{(0-6)^2 + (5-0)^2} = \sqrt{36+25} = \sqrt{61}$

(c) Here R is the mid-point of $\overline{PQ}$

$R\left(\frac{6+0}{2}, \frac{0+5}{2}\right) \Rightarrow R\left(3, \frac{5}{2}\right)$

(d) Equation of $\overline{OR}$ with $O(0,0) = (x_1, y_1)$ and $R\left(3, \frac{5}{2}\right) = (x_2, y_2)$

in two points form

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{\frac{5}{2} - 0}{3 - 0}(x - 3)$

$3y = \frac{5}{2}(x - 3)$

$5x - 6y - 15 = 0$

Q7. Let Saira bought x number of bananas and y number of apples.

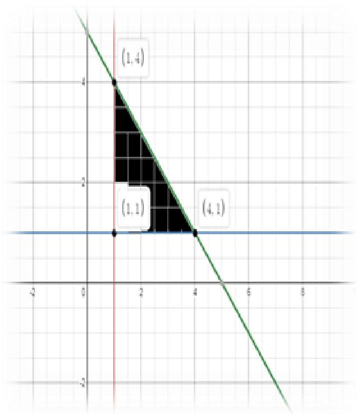
Price of one banana is Rs.6 and price of one apple is Rs.10

(a) According to the given conditions

$x \geq 1$, $y \geq 1$, $x + y \leq 5$ are the three constraints(inequalities).

(b) Graph is shown

(c) Shaded region is a triangle in the first quadrant with vertices as corner points A(1,1), B(4,1) and C(1,4)



d) Profit on one banana = $Rs. 26$

Profit on x bananas = $Rs. 26x$

Profit on one apple = $Rs. 10$

Profit on y apples = $Rs. 10y$

Objective Function: $P(x, y) = 26x + 10y$

Corner points	$P(x,y) = 26x + 10y$
$A(1,1)$	$P(1,1) = 26(1) + 10(1) = 36$
$B(4.1)$	$P(4,1) = 26(4) + 10(1) = 114$
$C(1,4)$	$P(1,4) = 26(1) + 10(4) = 66$

Hence to give shopkeeper maximum profit, Saira must buy 4 bananas and one apple.
The maximum profit to the shopkeeper is Rs 114

Q8. Let $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ be the required vertical Hyperbola.

Given that: $Sum\ of\ the\ axis\ = 32$

$$2a + 2b = 32$$

$$a + b = 16$$ --------eqn-I

Given that: eccentricity $e = \sqrt{7}$

$$\because b^2 = a^2(e^2 - 1)$$

$$b^2 = a^2\left(\left(\sqrt{7}\right)^2 - 1\right)$$

$$b^2 = 6a^2$$

$$b = \sqrt{6}a$$ ----------eqn-II

Substituting b's value in en-I

$$a + \sqrt{6}a = 16$$

$$a = 4.64$$

Substituting a's value in en-II

$$b = \sqrt{6}(4.64) = 11.37$$

Required Hyperbola: $\frac{x^2}{4.64^2} - \frac{y^2}{11.37^2} = 1$

Equation of the asymptotes: $y = \pm\frac{a}{b}x$

$$y = \frac{4.64}{11.37}x$$