

Version No.			

ROLL NUMBER						

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

0	0	0	0	0	0	0
1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9

Answer Sheet  
No. \_\_\_\_\_

Sign. of  
Candidate \_\_\_\_\_

Sign. of  
Invigilator \_\_\_\_\_

## MATHEMATICS HSSC-II

SECTION – A (Marks 20)

Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

**Q.1 Fill the relevant bubble for each part. All parts carry one mark.**

- What result occurs, in evaluating  $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$ ?
 

A. 9	<input type="radio"/>	B. -9	<input type="radio"/>
<b>C. 27</b>	<input checked="" type="radio"/>	D. does not exist	<input type="radio"/>
- Which one of the following represents an Odd function?
 

<b>A. <math>f(x) = \frac{3x}{x^2+1}</math></b>	<input checked="" type="radio"/>	B. $f(x) = 3x^4 - 2x^2 + 7$	<input type="radio"/>
C. $f(x) = \sin x + \cos x$	<input type="radio"/>	D. $f(x) = (x + 2)^2$	<input type="radio"/>
- Which one of the following represents  $f^{-1}(\sqrt{2})$ , if  $f(x) = \sqrt{2}\tan x$ ?
 

<b>A. <math>\frac{\pi}{4}</math></b>	<input checked="" type="radio"/>	B. $\frac{7\pi}{20}$	<input type="radio"/>
C. $\frac{\pi}{2}$	<input type="radio"/>	D. $\frac{3\pi}{4}$	<input type="radio"/>
- If  $f(x) = \cos x$ ,  $x \in \left(\frac{\pi}{2}, \pi\right)$  then what is the result of  $f'\left(\frac{3\pi}{4}\right)$ ?
 

A. $\frac{\sqrt{3}}{2}$	<input type="radio"/>	B. $\frac{1}{\sqrt{2}}$	<input type="radio"/>
C. $-\frac{\sqrt{3}}{2}$	<input type="radio"/>	<b>D. <math>-\frac{1}{\sqrt{2}}</math></b>	<input checked="" type="radio"/>
- In which one of the following intervals,  $f(x) = 2x^2 - 8x + 1$  increases its value?
 

A. $(-\infty, 2]$	<input type="radio"/>	B. $(-\infty, 0]$	<input type="radio"/>
C. $[0, \infty)$	<input type="radio"/>	<b>D. <math>(2, \infty)</math></b>	<input checked="" type="radio"/>
- For a function  $f(x) = \sin(\sin x)$  what evaluates  $f'(0)$ ?
 

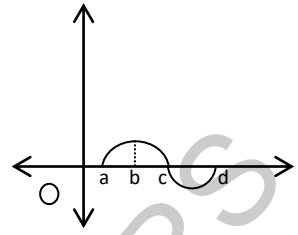
<b>A. 1</b>	<input checked="" type="radio"/>	B. 0	<input type="radio"/>
C. -1	<input type="radio"/>	D. does not exist	<input type="radio"/>
- Which one of the following options represents  $f'(x) = e^x + \sin x + 1$  and  $f(0) = 2$ ?
 

A. $f(x) = e^x + \cos x + x$	<input type="radio"/>
<b>B. <math>f(x) = e^x - \cos x + x + 2</math></b>	<input checked="" type="radio"/>
C. $f(x) = xe^{x-1} - \cos x + x + 3$	<input type="radio"/>
D. $f(x) = e^x + \cos x$	<input type="radio"/>

8. What results  $\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x} dx$ ?
- A.  $e - 1$   B.  $e$    
 C.  $\frac{\pi}{4}$   D.  $0$

9. The graph of  $f(x) = \int_a^x g(t)dt$  is shown in the given figure. For what value of  $x$ ,  $f(x)$  has its maximum value?

- A.  $a$   B.  $b$    
 C.  $c$   D.  $d$



10. Which one of the following lines passes through  $(-7, 7)$ ,  $(-7, -7)$  and  $(-7, 0)$ ?
- A.  $x = -7$   B.  $y = -7$    
 C.  $x + y = -7$   D.  $y = -x + 7$

11. How many intercepts are there in the graph of  $y = \frac{1}{x}$ ?
- A. no intercepts  B. two  $x$ -intercepts   
 C. two  $y$ -intercepts  D. one  $x$  and one  $y$ -intercept

12. At what angle lines  $3y = 2x + 5$  and  $3x + 2y = 8$  cut each other?
- A.  $\frac{\pi}{6}$   B.  $\frac{\pi}{4}$    
 C.  $0$   D.  $\frac{\pi}{2}$

13. Which one of the following options does not satisfy  $4x - 3y < 2$ ?
- A.  $(1, 1)$   B.  $(0, 0)$    
 C.  $(3, 0)$   D.  $(-2, 1)$

14. What are the coordinates of the centre of a circle  $x^2 + y^2 - 8x + 12y + 21 = 0$ ?
- A.  $(4, 6)$   B.  $(-4, 6)$    
 C.  $(4, -6)$   D.  $(-4, -6)$

15. What is the equation of the axis of a parabola  $y^2 - 2y + 8x - 23 = 0$ ?
- A.  $y = -1$   B.  $x = 3$    
 C.  $y = 1$   D.  $x = -3$

16. If  $(5, -2)$ ,  $(5, 4)$  are the vertices of a hyperbola, then centre of the hyperbola is:
- A.  $(0, 0)$   B.  $(5, 3)$    
 C.  $(5, 1)$   D.  $(5, 0)$

17. Which one of the following represents the graph of  $4x^2 + y^2 - 8x + 4y - 9 = 0$ ?
- A. circle  B. ellipse   
 C. parabola  D. hyperbola

18. For what value of  $\alpha$ , vectors  $4\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$  and  $\alpha\mathbf{i} + 3\mathbf{k}$  have the same magnitude?
- A.  $\pm 5$   B.  $5$    
 C.  $25$   D.  $-5$

19. If vectors  $3\mathbf{i} - 6\mathbf{j} + \mathbf{k}$  and  $2\mathbf{i} - 4\mathbf{j} + \lambda\mathbf{k}$  are parallel to each other, then the value of  $\lambda$  is:
- A.  $\frac{2}{3}$   B.  $\frac{3}{2}$    
 C.  $-\frac{3}{2}$   D.  $-\frac{2}{3}$

20. What is the projection of  $\mathbf{i} - \mathbf{k}$  along  $\mathbf{j} + \mathbf{k}$ ?
- A.  $\frac{1}{\sqrt{2}}$   B.  $-\frac{1}{2}$    
 C.  $-\frac{1}{\sqrt{2}}$   D.  $-1$

Federal Board HSSC-II Examination  
Mathematics Model Question Paper  
(Curriculum 2002)

**SECTION-A (1×20 = 20 Marks)**

Answer 1 Each part carries (01) mark. Key for MCQs is as under.

i	C	ii	A	iii	A	iv	D	v	D	vi	A	vii	B	viii	A	ix	B	x	A
xi	A	xii	D	xiii	C	xiv	C	xv	C	xvi	C	xvii	B	xviii	A	xix	A	xx	C

**SECTION-B ( 4×12 = 48 Marks)**

Answer 2 Attempt any TWELVE parts. Each part carries (04) marks.

- (i)  $f(x) = -4 + \sqrt{3-x}$  ,  $g(x) = \sqrt{x}$
- (a)  $f \circ g(x) = f[g(x)] = f(\sqrt{x}) = -4 + \sqrt{3-\sqrt{x}}$  (01 mark)
- (b)  $g \circ f(x) = g[f(x)] = g(-4 + \sqrt{3-x}) = \sqrt{-4 + \sqrt{3-x}}$  (01 mark)
- (c)  $f \circ f(x) = f[f(x)] = f(-4 + \sqrt{3-x}) = -4 + \sqrt{7 - \sqrt{3-x}}$  (01 mark)
- (d)  $g \circ g(x) = g[g(x)] = g(\sqrt{x}) = \sqrt{\sqrt{x}}$  (01 mark)
- (ii) (a)  $f(x) = -4 + \sqrt{3-x}$   
 Domain( $f$ ) =  $(-\infty, 3]$  = Range( $f^{-1}$ ) (01 mark)  
 Range( $f$ ) =  $[-4, +\infty)$  = Domain( $f^{-1}$ ) (01 mark)
- (b)  $f(x) = \frac{7+x}{x-1}$  ,  $x \neq 1$   
 Domain( $f$ ) =  $\mathcal{R} - \{1\}$  = Range( $f^{-1}$ ) (01 mark)  
 Range( $f$ ) =  $\mathcal{R} - \{1\}$  = Domain( $f^{-1}$ ) (01 mark)
- (iii)  $f(x) = (x^4 - x^3 + x^2 - x + 1)(3x^3 - 2x^2 + x - 1)$   
 Differentiating w.r.t. 'x'  
 $f'(x) = (x^4 - x^3 + x^2 - x + 1)[9x^2 - 4x + 1] + (3x^3 - 2x^2 + x - 1)[4x^3 - 3x^2 + 2x - 1]$  (02 marks)  
 $f'(1) = (1 - 1 + 1 - 1 + 1)[9 - 4 + 1] + (3 - 2 + 1 - 1)[4 - 3 + 2 - 1]$  (01 mark)  
 $f'(1) = (1)[6] + (1)[2] = 8$  (01 mark)
- (iv)  $x = 3 + \cos t$  ;  $y = 1 - \sin t$   
 Differentiating w.r.t. 't'  
 $\frac{dx}{dt} = -\sin t$   $\frac{dy}{dt} = -\cos t$  (01 + 01) marks  
 $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\cos t}{-\sin t} = \cot t$  (01 mark)
- (v)  $f(x) = (x^2 - 6x + 8)(x - 5)$   
 $f(x) = x^3 - 11x^2 + 38x - 40$   
 Differentiating w.r.t. x  
 $f'(x) = 3x^2 - 22x + 38$  (01 mark)  
 At extreme  $f'(x) = 0$   
 $x^2 - 22x + 38 = 0 \Rightarrow x = \frac{22 \pm \sqrt{484 - 456}}{2} = \frac{11 \pm \sqrt{7}}{1} = 2.8 \text{ or } 4.5$  (01 mark)  
 Consider  $f'(2) = 3(2)^2 - 22(2) + 38 = 6 > 0 \Rightarrow f$  increases in  $(-\infty, 2.8)$   
 Consider  $f'(3) = 3(3)^2 - 22(3) + 38 = -1 < 0 \Rightarrow f$  decreases in  $(2.8, 4.5)$   
 Consider  $f'(5) = 3(5)^2 - 22(5) + 38 = 3 > 0 \Rightarrow f$  increases in  $(4.5, +\infty)$   
 $f$  increases in  $(-\infty, 2.8) \cup (4.5, +\infty)$  and decreases in  $(2.8, 4.5)$  (01 + 01) marks
- (vi) Let  $f(x) = x^{1/5}$  with  $x = 32$  and  $\delta x = dx = 1$  (01 mark)  
 Differentiating w.r.t. 'x'  
 $f'(x) = \frac{1}{5} x^{-4/5}$  (01 mark)  
 $\therefore f(x + \delta x) = f(x) + f'(x)dx$

$$f(x + \delta x) = x^{1/5} + \frac{1}{5}x^{-4/5}dx \quad (01 \text{ mark})$$

$$f(32 + 1) = (32)^{1/5} + \frac{1}{5}(32)^{-4/5}(1)$$

$$f(33) = 2 + \frac{1}{80}$$

$$33^{1/5} = \frac{161}{80} \quad (01 \text{ mark})$$

$$(vii) \int \frac{\ln x}{x^2} dx = \int (\ln x)(x)^{-2} dx \quad (01 \text{ mark})$$

$$= (\ln x) \left(-\frac{1}{x}\right) - \int \left(\frac{1}{x}\right) \left(-\frac{1}{x}\right) dx \quad (01 \text{ mark})$$

$$= -\frac{1}{x} \ln x + \int x^{-2} dx \quad (01 \text{ mark})$$

$$= -\frac{1}{x} \ln x - \frac{1}{x} + c \quad (01 \text{ mark})$$

$$(viii) f(x) = 4x - x^2$$

$$\text{For } x\text{-intercepts } 4x - x^2 = 0 \Rightarrow x(4 - x) = 0 \Rightarrow x = 0, 4 \quad (01 \text{ mark})$$

$$\text{Required Area} = \int_0^4 f(x) = \int_0^4 (4x - x^2) dx \quad (01 \text{ mark})$$

$$= \left[ \frac{4x^2}{2} - \frac{x^3}{3} \right]_0^4 = \frac{1}{3} [6x^2 - x^3]_0^4 \quad (01 \text{ mark})$$

$$= \frac{32}{3} \text{ Square units} \quad (01 \text{ mark})$$

$$(ix) \text{ Let } l \text{ be the required line having slope } m = \tan 30^\circ = \frac{1}{\sqrt{3}} \quad (01 \text{ mark})$$

$$\text{Equation of the line } l \text{ passing through } (x_1, y_1) = (-4, 8) \text{ with slope } m = \frac{1}{\sqrt{3}} \text{ is}$$

$$y - y_1 = m(x - x_1) \quad (01 \text{ mark})$$

$$y - 8 = \frac{1}{\sqrt{3}}(x + 4) \quad (01 \text{ mark})$$

$$x - \sqrt{3}y + 8\sqrt{3} + 4 = 0 \quad (01 \text{ mark})$$

$$(x) \text{ Let } l : 2x - 4y + 7 = 0 \text{ be the given line}$$

$$\text{Now } -2x + 4y - 7 = 0$$

$$\text{At } P(3, 1)$$

$$-2x + 4y - 7 = -2(3) + 4(1) - 7 = -9 < 0 \quad (01 \text{ mark})$$

$$P \text{ lies below } l \quad (01 \text{ mark})$$

$$\text{At } Q(-1, 6)$$

$$-2x + 4y - 7 = -2(-1) + 4(6) - 7 = 20 > 0 \quad (01 \text{ mark})$$

$$Q \text{ lies above } l \quad (01 \text{ mark})$$

$$(xi) \text{ Constraints: } 10x + 20y \leq 140 ; 6x + 18y \geq 72 ; x \geq 0 ; y \geq 0$$

$$\text{Corresponding Equations: } 10x + 20y = 140 ; \quad 6x + 18y = 72$$

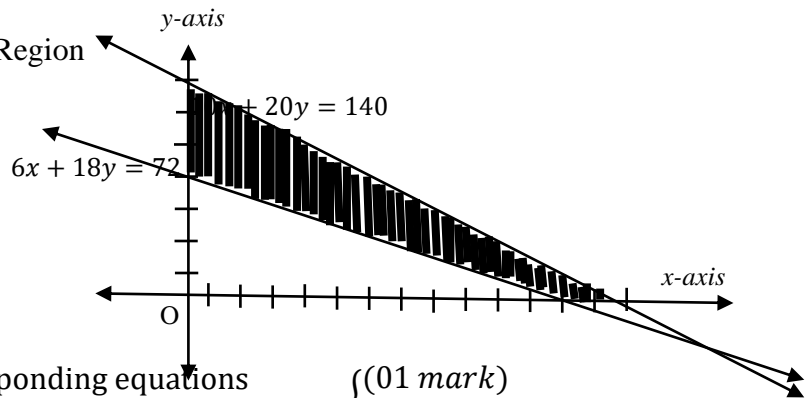
$$\text{Intercepts: } (14, 0), (0, 7) \quad (12, 0), (0, 4) \quad (01 \text{ mark})$$

$$\text{In Equations: } 10x + 20y < 140 ; \quad 6x + 18y > 72$$

$$\text{Test Point: } O(0, 0) \quad 0 < 140 \quad 0 > 72 \quad (01 \text{ mark})$$

$$\text{Solution Region lies towards Test Point side opposite to Test Point side}$$

Common shaded region is the Solution Region



Graph: { Lines representing the corresponding equations } (01 mark)  
 { Correct shade of the feasible solution region } (01 mark)

$$(xii) \text{ Let } l_1: 3y = 4x - 5 \text{ and } l_2: 3y = -4x - 13$$

$$l_1 + l_2: 6y = -18 \Rightarrow y = -3$$

$$l_1 - l_2: 0 = 8x + 8 \Rightarrow x = -1$$

$$\text{Centre of the circle: } C(h, k) = (-1, -3) \quad (01 \text{ mark})$$

If  $P(-5, 0)$  be the point lying on the circle,

$$\text{then radius } r = |CP| = \sqrt{(-5 + 1)^2 + (0 + 3)^2} = 5 \quad (01 \text{ mark})$$

$$\text{Circle Equation: } (x - h)^2 + (y - k)^2 = r^2 \quad (01 \text{ mark})$$

$$(x + 1)^2 + (y + 3)^2 = 5^2$$

$$\text{or } x^2 + y^2 + 2x + 6y - 15 = 0 \quad (01 \text{ mark})$$

(xiii) Let  $F(-2, 1)$  be the focus and  $l: x - 5 = 0$  be the directrix of the parabola.

Consider a point  $P(x, y)$  on the parabola. Draw  $\overline{PM} \perp l$  and join  $P$  to  $F$ .

By definition  $|\overline{PF}| = |\overline{PM}|$  (01 mark)

$$\sqrt{(x+2)^2 + (y-1)^2} = \frac{|x-5|}{\sqrt{1^2}} \quad (01 \text{ mark})$$

$$(x+2)^2 + (y-1)^2 = (x-5)^2 \quad (01 \text{ mark})$$

$$(y-1)^2 = 7(3-2x) \quad (01 \text{ mark})$$

(xiv)  $16x^2 + 25y^2 = 1 \Rightarrow \frac{x^2}{1/16} + \frac{y^2}{1/25} = 1$

Comparing with  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  to get  $a^2 = 1/16$  and  $b^2 = 1/25$

At the point  $(x_1, y_1) = (4, 12/5)$

Equation of the tangent:  $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$  (01 mark)

$$\Rightarrow \frac{4x}{1/16} + \frac{12y/5}{1/25} = 1 \Rightarrow 64x + 60y - 1 = 0 \quad (01 \text{ mark})$$

Equation of the normal:  $y - y_1 = \frac{a^2 y_1}{b^2 x_1} (x - x_1)$  (01 mark)

$$\Rightarrow y - \frac{12}{5} = \frac{(\frac{1}{16})(\frac{12}{5})}{(\frac{1}{25})^4} (x - 4) \Rightarrow 75x - 80y - 108 = 0 \quad (01 \text{ mark})$$

(xv)  $\underline{u} = -2\underline{i} + 5\underline{j} + 3\underline{k}$  ;  $\underline{v} = \underline{i} + 3\underline{j} - 2\underline{k}$  ;  $\underline{w} = -3\underline{i} + \underline{j} - 2\underline{k}$

Volume of Parallelepiped =  $\begin{vmatrix} -2 & 5 & 3 \\ 1 & 3 & -2 \\ -3 & 1 & -2 \end{vmatrix}$  (02 marks)

$$= -2(-4) - 5(-8) + 3(10) = 78 \quad (01 + 01) \text{ marks}$$

(xvi)  $\underline{u} = 3\underline{i} + \underline{j} - \underline{k}$  ;  $\underline{v} = 2\underline{i} - \underline{j} + \underline{k}$

$$\underline{u} \cdot \underline{v} = (3)(2) + (1)(-1) + (-1)(1) = 4 \quad (01 \text{ mark})$$

$$|\underline{u}| = \sqrt{9 + 1 + 1} = \sqrt{11} ; |\underline{v}| = \sqrt{4 + 1 + 1} = \sqrt{6} \quad (01 \text{ mark})$$

If  $\theta$  be the angle between  $\underline{u}$  and  $\underline{v}$ , then

$$\cos \theta = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}| |\underline{v}|} = \frac{4}{\sqrt{11} \sqrt{6}} \quad (01 \text{ mark})$$

$$\theta = \cos^{-1} \left( \frac{4}{\sqrt{66}} \right) = 60.5^\circ \quad (01 \text{ mark})$$

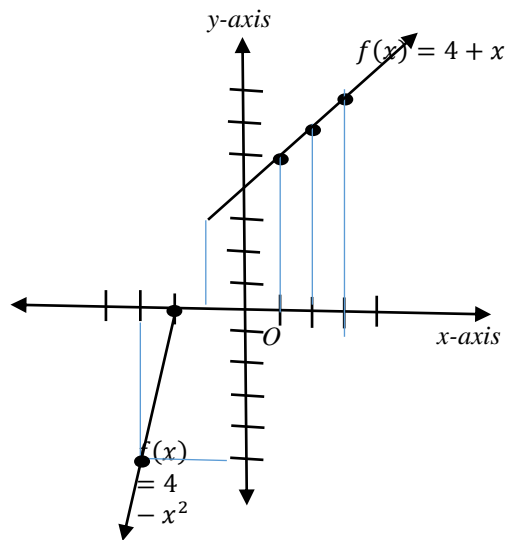
**SECTION-C ( 8x4 = 32 Marks )**

Answer 3  $f(x) = \begin{cases} 4 - x^2, & \text{if } x \leq 0 \\ 4 + x, & \text{if } x > 0 \end{cases}$

(a) Correct Table of values (01 mark)

Correct graph (01 mark)

x	-3	-2	-1	0	1	2	3
y	-5	0	3	4	5	6	7



(b) At  $x = 0$ ,  $f(x) = 4 - x^2$   
 $\Rightarrow f(0) = 4$  (01 mark)

(c)  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (4 - x^2) = 4$  (01 mark)

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (4 + x) = 4 \quad (01 \text{ mark})$$

(d) Since  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = 4$   
 $\Rightarrow \lim_{x \rightarrow 0} f(x) = 4$  (01 mark)

And  $\lim_{x \rightarrow 0} f(x) = f(0)$  (01 mark)

Therefore  $f$  is continuous at  $x = 0$  (01 mark)

Answer 4  $f(x) = \sin x + \cos^2 x$  ;  $x \in \left[0, \frac{\pi}{2}\right]$

(a) Differentiating w.r.t.  $x$   
 $f'(x) = \cos x - 2\cos x \cdot \sin x$  (01 mark)

(b) Differentiating again w.r.t.  $x$   
 $f''(x) = -\sin x - 2(\cos^2 x - \sin^2 x) = -\sin x - 2\cos 2x$  (01 mark)

- (c) For extreme values put  $f'(x) = 0$   
 $\cos x - 2\cos x \cdot \sin x = 0 \Rightarrow \cos x(1 - 2\sin x) = 0$   
 $\cos x = 0$  gives  $x = \frac{\pi}{2}$  (01 mark)  
 $1 - 2\sin x = 0$  gives  $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$  (01 mark)
- (d)  $f''\left(\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) - 2\cos 2\left(\frac{\pi}{2}\right) = 1 > 0$  (01 mark)  
 $f_{\min} = f\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) + \cos^2\left(\frac{\pi}{2}\right) = 1$  (01 mark)  
 $f''\left(\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) - 2\cos 2\left(\frac{\pi}{6}\right) = -\frac{3}{2} < 0$  (01 mark)  
 $f_{\max} = f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{6}\right) = \frac{5}{4}$  (01 mark)

Answer 5

- $$\int \frac{x^3+4}{(x^2-1)(x^2+3x+2)} dx$$
- (a)  $\frac{x^3+4}{(x+1)^2(x-1)(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{x+2} \rightarrow \text{eqn I}$   
 $x^3 + 4 = A(x+1)(x-1)(x+2) + B(x-1)(x+2) + C(x+1)^2(x+2) + D(x+1)^2(x-1) \rightarrow \text{eqn II}$   
For B put  $x = -1$   $-1 + 4 = B(-2)(1) \Rightarrow B = -3/2$  (01 mark)  
For C put  $x = 1$   $1 + 4 = C(4)(3) \Rightarrow C = 5/12$  (01 mark)  
For D put  $x = -2$   $-8 + 4 = D(1)(-3) \Rightarrow D = 4/3$  (01 mark)  
Consider eqn II as  
 $x^3 + 4 = A(x^3 + 2x^2 - x - 2) + B(x^2 + x - 2) + C(x^3 + 4x^2 + 5x + 2) + D(x^3 + x^2 - x - 1)$   
Equating the coefficients of like powers of  $x^3$   
 $1 = A + C + D$   
 $1 = A + \frac{5}{12} + \frac{4}{3} \Rightarrow A = -\frac{3}{4}$  (01 mark)  
Substituting values in eqn I  
 $\frac{x^3+4}{(x+1)^2(x-1)(x+2)} = -\frac{3}{4(x+1)} - \frac{3}{2(x+1)^2} + \frac{5}{12(x-1)} + \frac{4}{3(x+2)}$
- (b)  $\int \frac{x^3+4}{(x+1)^2(x-1)(x+2)} dx = -\frac{3}{4} \int \frac{1}{x+1} dx - \frac{3}{2} \int (x+1)^{-2} dx + \frac{5}{12} \int \frac{1}{x-1} dx + \frac{4}{3} \int \frac{1}{x+2} dx$   
 $\int \frac{x^3+4}{(x+1)^2(x-1)(x+2)} dx = -\frac{3}{4} \ln|x+1| + \frac{3}{2(x+1)} + \frac{5}{12} \ln|x-1| + \frac{4}{3} \ln|x+2|$   
(01 + 01 + 01 + 01)marks

Answer 6

- $A(-1, 1), B(5, 5), C(4, 1)$
- (a) Slope of  $l = m = \frac{5-1}{5+1} = \frac{2}{3}$  (01 mark)
- (b) Equation of  $l$  through point  $(x_1, y_1) = A(-1, 1)$   
is given by  $y - y_1 = m(x - x_1)$  (01 mark)  
 $y - 1 = \frac{2}{3}(x + 1) \Rightarrow 2x - 3y + 5 = 0$  (01 mark)
- (c)  $2x - 3y + 5 = 0 \Rightarrow -2x + 3y - 5 = 0$   
Divide the eqn by  $\sqrt{(2)^2 + (-3)^2}$  or  $\sqrt{13}$   
 $-\frac{2}{\sqrt{13}}x + \frac{3}{\sqrt{13}}y = \frac{5}{\sqrt{13}}$  (01 mark)  
Comparing with  $x\cos\alpha + y\sin\alpha = p$   
 $\cos\alpha = -\frac{2}{\sqrt{13}} < 0$  ;  $\sin\alpha = \frac{3}{\sqrt{13}} > 0$  ;  $p = \frac{5}{\sqrt{13}}$  ( $\alpha$  in 2nd Quadrant)  
 $\Rightarrow \tan\alpha = -\frac{3}{2} \alpha = \tan^{-1}\left(-\frac{3}{2}\right) = 123.69^\circ$  (01 mark)  
Normal Form:  $x\cos(123.69^\circ) + y\sin(123.69^\circ) = \frac{5}{\sqrt{13}}$  (01 mark)
- (d)  $A(-1, 1), B(5, 5), C(4, 1)$   
Area of Triangle ABC =  $\frac{1}{2} \begin{vmatrix} -1 & 1 & 1 \\ 4 & 1 & 1 \\ 5 & 5 & 1 \end{vmatrix}$  (01 mark)  
=  $\frac{1}{2}[-1(-4) - 1(-1) + 1(15)] = 10$  (01 mark)

Answer 7

- Let number of conventional phones be  $x$  and number of smart phones be  $y$ .  
Time constraints for assembling and finishing of conventional and smart phones are  
 $x + 2y \leq 24$  and  $2x + y \leq 24$  respectively.  
The restriction on number of gadgets in a day is  $x + y \leq 15$   
The objective function as Profit function is  $P(x, y) = 1000x + 4000y$   
Constraints:  $x + 2y \leq 24$  ;  $2x + y \leq 24$  ;  $\geq 0$  ;  $y \geq 0$   
Corresponding Equations:  $x + 2y = 24$  ;  $2x + y = 24$   
Intercepts:  $(24, 0), (0, 12)$ ;  $(12, 0), (0, 24)$  (01 mark)  
In Equations:  $x + 2y < 24$  ;  $2x + y < 24$   
Test Point:  $O(0, 0)$   $0 < 24; 0 < 24$  (01 mark)  
Solution Region lies: towards Test Point side

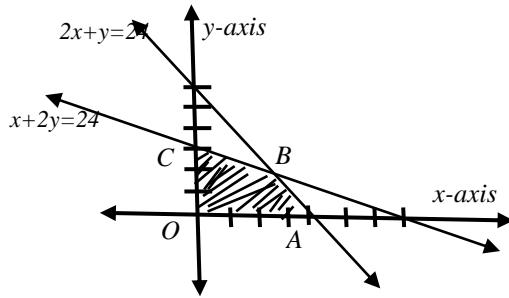
The graph shows OABC as feasible solution region.

For B solving  $x + 2y = 24$  and  $2x + y = 24$

$$\frac{x}{-48+24} = \frac{-y}{-24+48} = \frac{1}{1-4} \Rightarrow x = 8, y = 8 \Rightarrow B(8, 8) \quad (01 \text{ mark})$$

Graph:  $\left\{ \begin{array}{l} \text{Lines representing the corresponding equations} \\ \text{Correct shade of the feasible solution region} \end{array} \right. \quad \left\{ \begin{array}{l} (01 + 01) \text{ marks} \\ (01 \text{ mark}) \end{array} \right.$

Corner Points	Objective Function $P(x, y) = 1000x + 4000y$
$O(0, 0)$	$P(0, 0) = 0$
$A(12, 0)$	$P(12, 0) = 12,000$
$B(8, 8)$	$P(8, 8) = 40,000$
$C(0, 12)$	$P(0, 12) = 48,000$ (max)



Correct Table of values

(01 mark)

The point  $C(0, 12)$  gives the most profit, and that profit is Rs. 48,000.

Therefore, we conclude that one should manufacture 12 smart phones daily to obtain the maximum profit. (01 mark)

Answer 8  $4x^2 - 5y^2 + 40x - 30y - 45 = 0$

$$4(x + 5)^2 - 5(y + 3)^2 = 100$$

$$\frac{(x+5)^2}{25} - \frac{(y+3)^2}{20} = 1 \quad (\text{Horizontal Hyperbola}) \quad (01 \text{ mark})$$

$$\frac{X^2}{25} - \frac{Y^2}{20} = 1 \quad \text{where } X = x + 5 \text{ and } Y = y + 3$$

$$\text{Comparing with } \frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1 \text{ to get } a = 5 \text{ and } b = 2\sqrt{5} \quad (01 \text{ mark})$$

$$\text{Taking } c^2 = a^2 + b^2 = 25 + 20 = 45 \Rightarrow c = 3\sqrt{5} \quad (01 \text{ mark})$$

$$\begin{aligned} \text{Centre: } (0, 0) &\Rightarrow X = 0 ; Y = 0 \\ &\Rightarrow x + 5 = 0 ; y + 3 = 0 \\ &\Rightarrow x = -5 ; y = -3 \Rightarrow (-5, -3) \quad (01 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{Foci: } (\pm c, 0) &\Rightarrow X = \pm c ; Y = 0 \\ &\Rightarrow x + 5 = \pm 3\sqrt{5} ; y + 3 = 0 \\ &\Rightarrow x = -5 \pm 3\sqrt{5} ; y = -3 \Rightarrow (-5 \pm 3\sqrt{5}, -3) \quad (01 \text{ mark}) \end{aligned}$$

$$\text{Eccentricity: } e = \frac{c}{a} = \frac{3\sqrt{5}}{5} \quad (01 \text{ mark})$$

$$\begin{aligned} \text{Vertices: } (\pm a, 0) &\Rightarrow X = \pm a ; Y = 0 \\ &\Rightarrow x + 5 = \pm 5 ; y + 3 = 0 \\ &\Rightarrow x = -5 \pm 5 ; y = -3 \Rightarrow (-5 \pm 5, -3) \quad (01 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{Directrices: } X &= \pm \frac{c}{e^2} \\ \Rightarrow x + 5 &= \pm \frac{3\sqrt{5}}{(3\sqrt{5}/5)^2} \Rightarrow x = \frac{-15 \pm 5\sqrt{5}}{3} \quad (01 \text{ mark}) \end{aligned}$$