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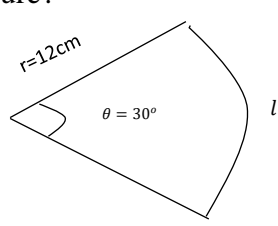Answer Sheet No._____

Sign. of Candidate _____

Sign. of Invigilator _____

MATHEMATICS HSSC–I (2nd Set)
SECTION – A (Marks 20)
Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

Q.1 **Fill the relevant bubble for each part. All parts carry one mark.**

(1) If $\frac{2}{1-i} - x = 0$, then value of x is:
   A. $-1 - i$ ◯
   B. $-1 + i$ ◯
   C. $1 - i$ ◯
   D. $1 + i$ ◯

(2) If A and B are two sets and $A \cap B = \phi$, then $n(A \cup B)$ is:
   A. $n(A) + n(B)$ ◯
   B. $n(A)$ ◯
   C. $n(B)$ ◯
   D. $n(A) + n(B) - n(A \cap B)$ ◯

(3) If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then what is the order of matrix X?
   A. 2×2 ◯
   B. 2×3 ◯
   C. 3×2 ◯
   D. 3×3 ◯

(4) If $x^3 + 3x^2 - 6x + 2$ is divided by $x + 2$, then the remainder is:
   A. -18 ◯
   B. 9 ◯
   C. -9 ◯
   D. 18 ◯

(5) If α, β are the roots of the equation $3x^2 - 2x - 9 = 0$, then $(\alpha + 1)(\beta + 1)$ is:
   A. $-\frac{2}{3}$ ◯
   B. $\frac{2}{3}$ ◯
   C. $-\frac{1}{3}$ ◯
   D. $\frac{1}{3}$ ◯

(6) For how many values of x, the expression $x^2 - x - 2 = (x + 1)(x - 2)$ holds?
   A. For no value of x ◯
   B. For only one value of x ◯
   C. For only two values of x ◯
   D. For all values of x ◯

Page 1 of 3

(7) The series $1 + \frac{x}{2} + \frac{x^2}{2} + \cdots$ is convergent if:

A. $x \in R$ ○   B. $x \in [-2, 2]$ ○

C. $x \in (-2, 2)$ ○   **D. $x \in Z$** ○

(8) Which of the following series represents $\sum\limits_{n=1} 6(3)^{n-1}$?

A. $6 + 9 + 12 + \cdots$ ○   **B. $6 + 18 + 54 + \cdots$** ○

C. $3 + 9 + 27 + \cdots$ ○   D. $6 + 12 + 18 + \cdots$ ○

(9) The probability of getting a total of 10 in a single throw of two dice is:

A. $\frac{1}{9}$ ○   B. $\frac{1}{12}$ ○

C. $\frac{1}{6}$ ○   D. $\frac{5}{36}$ ○

(10) In how many ways can we choose a committee of 5 from 8 persons?

A. 56 ○   B. 336 ○

C. 6720 ○   D. 6 ○

(11) The middle term in the expansion of $(a + b)^6$ is:

A. T_3 ○   **B. T_4** ○

C. T_5 ○   D. T_6 ○

(12) The expansion of $(1 - 2x)^{\frac{1}{3}}$ is valid if

A. $|x| > \frac{1}{2}$ ○   B. $|x| > 1$ ○

C. $|x| < \frac{1}{2}$ ○   **D. $|x| < 2$** ○

(13) What is the value of l in the adjoining figure?

A. π ○

B. 2π ○

C. 3π ○

D. 4π ○

(14) $\sin 294° = $ _____.

A. $\sin 24^o$ ○   **B. $\cos 24^o$** ○

C. $-\sin 24^o$ ○   D. $-\cos 24^o$ ○

(15) Which one of the following is equal to $\cos(\alpha + \beta)$ if $\alpha + \beta + \gamma = 180^o$?

A. $\sin\gamma$ ○   **B. $\cos\gamma$** ○

C. $-\cos\gamma$ ○   D. $-\sin\gamma$ ○

(16) At what angle, the graph of $y = \cos 2x$ crosses $x -$axis?

A. $\frac{\pi}{4}$ ○   B. $\frac{\pi}{2}$ ○

C. π ○   **D. 0** ○

(17) If $a = 2$, $b = 3$ and $\gamma = 30^o$, then triangular area is:

A. 1.5 ○   **B. 0.8** ○

C. 2.6 ○   D. 2.1 ○

Section – A

1	C	2	B	3	C	4	B	5	A	6	B	7	D	8	B	9	A	10	A
11	B	12	D	13	C	14	B	15	B	16	D	17	B	18	A	19	B	20	D

SECTION-B

Q2.

(i)   If $z = \sqrt{2} - i$ then $\bar{z} = \sqrt{2} + i$

a) $z^2 + \bar{z}^2 = (\sqrt{2} - i)^2 + (\sqrt{2} + i)^2$

$z^2 + \bar{z}^2 = 2 - 2\sqrt{2}i + i^2 + 2 + 2\sqrt{2}i + i^2$

$z^2 + \bar{z}^2 = 2 + (-1) + 2 + (-1) = 0 \in \mathcal{R}$

b) $(z - \bar{z})^2 = (\sqrt{2} - i - \sqrt{2} - i)^2$

$(z - \bar{z})^2 = (-2i)^2$

$(z - \bar{z})^2 = 4i^2 = -4 \in \mathcal{R}$

(ii) $p \rightarrow q = \sim (p \wedge \sim q)$

p	q	$\sim q$	$p \wedge \sim q$	$\sim (p \wedge \sim q)$	$p \rightarrow q$
T	T	F	**F**	T	**T**
T	F	T	**T**	F	**F**
F	T	F	**F**	T	**T**
F	F	T	**F**	T	**T**

From column 4 and column 6: $p \rightarrow q = \sim (p \wedge \sim q)$

(iii)　　$A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -2 & 2 \\ 1 & 2 & -3 \end{bmatrix}$

　　　a)　　$A_{11} = \begin{vmatrix} -2 & 2 \\ 2 & -3 \end{vmatrix} = 6 - 4 = 2$

　　　　　$A_{21} = \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} - 6 + 1 = -5$

　　　　　$A_{31} = \begin{vmatrix} 2 & -1 \\ -2 & 2 \end{vmatrix} = 4 + 2 = 6$

　　　b)　　$|A| = A_{11} - A_{21} + A_{31} = 2 + 5 + 6 = 13$

(iv)　　$y + 2 = 11x$　　$\implies$　　$y = 11x - 2$　.......... eqn-I

　　　$y = 25x^2 - 9x + 2$

　　　Using eqn-I

　　　$11x - 2 = 25x^2 - 9x + 2$

　　　$25x^2 - 20x + 4 = 0$

　　　$(5x - 2)^2 = 0$

　　　$x = \dfrac{2}{5}$

　　　Substitute value of x in eq-I

　　　$y = 11\left(\dfrac{2}{5}\right) - 2 = \dfrac{12}{5}$

　　　Solution Set $= \left\{\left(\dfrac{2}{5}, \dfrac{12}{5}\right)\right\}$

(v)　　$(x - p)(x - q) + (x - q)(x - r) + (x - r)(x - p) = 0$

　　　$x^2 - (p + q)x + pq + x^2 - (q + r)x + qr + x^2 - (r + p)x + rp = 0$

　　　$3x^2 - 2(p + q + r)x + (pq + qr + rp) = 0$

　　　Discriminant $= b^2 - 4ac$

　　　　　　$= [-2(p + q + r)]^2 - 4(3)(pq + qr + rp)$

　　　　　　$= 4(p^2 + q^2 + r^2 + 2pq + 2qr + 2rp - 3pq - 3qr - 3rp)$

　　　　　　$= 4(p^2 + q^2 + r^2 - pq - qr - rp)$

　　　　　　$= 2(2p^2 + 2q^2 + 2r^2 - 2pq - 2qr - 2rp)$

　　　　　　$= 2[p^2 + p^2 + q^2 + q^2 + r^2 + r^2 - 2pq - 2qr - 2rp]$

　　　　　　$= 2[p^2 + q^2 - 2pq + q^2 + r^2 - 2qr + r^2 + + p^2 - 2rp]$

　　　　　　$= 2[(p - q)^2 + (q - r)^2 + (r - p)^2] > 0$

　　i.e. roots of the given equation are real.

For equal roots

Discriminant $= 0$

$2[(p-q)^2 + (q-r)^2 + (r-p)^2] = 0$

$(p-q)^2 + (q-r)^2 + (r-p)^2 = 0$

This is only possible if

$(p-q)^2 = 0 \quad \Longrightarrow \quad p = q$

$(q-r)^2 = 0 \quad \Longrightarrow \quad q = r$

$(r-p)^2 = 0 \quad \Longrightarrow \quad r = p$

Hence $p = q = r$

(vi) $\quad \dfrac{2x-3}{(x^2-x+1)(3x-2)} = \dfrac{Ax+B}{(x^2-x+1)} + \dfrac{C}{(3x-2)}$----------eqn-I

$2x - 3 = (Ax + B)(3x - 2) + C(x^2 - x + 1)$---------eqn-II

For C put $3x - 2 = 0$ or $x = \dfrac{2}{3}$ in eqn-I

$2\left(\dfrac{2}{3}\right) - 3 = 0 + C\left(\dfrac{4}{9} - \dfrac{2}{3} + 1\right)$

$\dfrac{4}{3} - 3 = C\left(-\dfrac{2}{9} + 1\right) \qquad \Longrightarrow C = -\dfrac{15}{7}$

Simplifying eqn-II as

$2x - 3 = A(3x^2 - 2x) + B(3x - 2) + C(x^2 - x + 1)$

Equating the coefficients of x^2, constant terms

$0 = 3A + C \qquad\qquad\qquad -3 = -2B + C$

$0 = 3A - \dfrac{15}{7} \qquad\qquad\qquad -3 = -2B - \dfrac{15}{7}$

$A = -\dfrac{5}{7} \qquad\qquad\qquad\qquad B = \dfrac{3}{7}$

$Substituting\ the\ values\ in\ eqn - I$

$\dfrac{2x-3}{(x^2-x+1)(3x-2)} = \dfrac{-\frac{5}{7}x+\frac{3}{7}}{(x^2-x+1)} + \dfrac{-\frac{15}{7}}{(3x-2)}$

$\dfrac{2x-3}{(x^2-x+1)(3x-2)} = \dfrac{-5x+3}{7(x^2-x+1)} - \dfrac{15}{7(3x-2)}$

(vii) A.P: b, c, p, q, r

Consider the following

$a_1 + a_5 = b + r \qquad\qquad a_2 + a_4 = c + q \qquad\qquad 2a_3 = 2p$

$a + (a + 4d) = b + r \qquad\qquad a + d + a + 3d = c + q \qquad\qquad 2(a + 2d) = 2p$

$$2a + 4d = b + r \qquad\qquad 2a + 4d = c + q \qquad\qquad 2a + 4d = 2p$$

From above $\quad b + r = c + q = 2p$

(viii) In HP: $a_p = q$; $a_q = p$; $\quad a_{pq} = ?$

In AP: $a_p = \dfrac{1}{q}$; $a_q = \dfrac{1}{p}$

$\because a_n = a_1 + (n-1)d$

$$a_p = a_1 + (p-1)d \qquad\qquad a_q = a_1 + (q-1)d$$

$$\dfrac{1}{q} = a_1 + pd - d \text{ -----eqn-I} \qquad\qquad \dfrac{1}{p} = a_1 + qd - d$$

$$\dfrac{1}{q} - \dfrac{1}{p} = (a_1 + pd - d) - (a_1 + qd - d)d$$

$$\dfrac{1}{q} - \dfrac{1}{p} = (p-q)d$$

$$\dfrac{p-q}{pq} = (p-q)d$$

$$d = \dfrac{1}{pq}$$

Put the value of d in eqn-I giving $a_1 = \dfrac{1}{pq}$

$$a_{pq} = a_1 + (pq-1)d$$

$$a_{pq} = \dfrac{1}{pq} + (pq-1)\dfrac{1}{pq}$$

$$a_{pq} = \dfrac{1}{pq} + 1 - \dfrac{1}{pq} = 1$$

$$a_{pq} = 1$$

Thus $(pq)th$ term of H.P $= 1$

(ix) (a) when C and K are placed together

Number of permutations $= 2! \times 5! = 2 \times 120 = 240$

(b) Total number of permutation$= 6! = 720$

when C and K are not placed together

Number of permutations

$=$Total number of permutation $-$ Number of permutation when C and K are together

$= 720 - 240 = 480$

(x) Let, $a > b$ and $= b + h$, where h is so small that its square and higher power be neglected.

$$\text{L.H.S} = \frac{b+2a}{a+2b} = \frac{b+2(b+h)}{(b+h)+2b} = \frac{3b+2h}{3b+h} = \frac{1+\frac{2h}{3b}}{1+\frac{h}{3b}}$$

$$= \left(1+\frac{2h}{3b}\right)\left(1+\frac{h}{3b}\right)^{-1}$$

$$= \left(1+\frac{2h}{3b}\right)\left(1+(-1)\frac{h}{3b} + \frac{(-1)(-2)}{2!}\left(\frac{h}{3b}\right)^2 + \cdots\right)$$

$$\approx \left(1+\frac{2h}{3b}\right)\left(1-\frac{h}{3b}\right)$$

$$\approx 1 - \frac{h}{3b} + \frac{2h}{3b}$$

$$\text{L.H.S} \approx 1 + \frac{h}{3b}$$

$$\text{R.H.S.} = \sqrt[3]{\frac{a}{b}} = \left(\frac{b+h}{b}\right)^{\frac{1}{3}} = \left(1+\frac{h}{b}\right)^{\frac{1}{3}} \approx 1+\frac{h}{3b}$$

Hence L.H.S = R.H.S

(xi) In the given right triangle where θ lies in the first quadrant

Let $= 12$, $y = 5$, $r = 13$

$$\sec\theta = \frac{r}{x} = \frac{13}{12} \qquad \csc\theta = \frac{r}{y} = \frac{13}{5} \qquad \tan\theta = \frac{y}{x} = \frac{5}{12} \qquad \cot\theta = \frac{x}{y} = \frac{12}{5}$$

a) $\sec^2\theta - \tan^2\theta = \left(\frac{13}{12}\right)^2 - \left(\frac{5}{12}\right)^2 = \frac{169}{144} - \frac{25}{144} = \frac{144}{144} = 1$

b) $\csc^2\theta - \cot^2\theta = \left(\frac{13}{5}\right)^2 - \left(\frac{12}{5}\right)^2 = \frac{169}{25} - \frac{144}{25} = \frac{25}{25} = 1$

(xii) $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

$$L.H.S = \tan(\alpha - \beta)$$

$$= \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$$

$$= \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

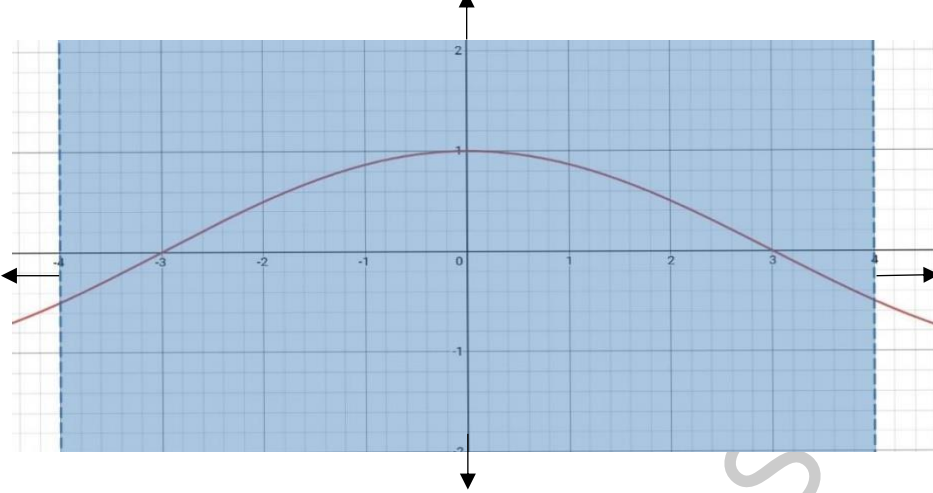Dividing the numerator and denominator by $\cos\alpha\cos\beta$

$$= \frac{\left(\frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta}\right)}{\left(\frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta}\right)}$$

$$= \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

$$= \text{R.H.S.}$$

(xiii) $\quad y = \cos\left(\frac{\pi}{6}\, x\right) \qquad -4 \le x \le 4$

x	−4	−3	−2	−1	0	1	2	3	4
y	−0.5	0	0.5	0.87	1	0.87	0.5	0	0.5

(xiv) $\quad L.H.S = \dfrac{\cos\alpha}{a} + \dfrac{\cos\beta}{b} + \dfrac{\cos\gamma}{c}$

By Law of cosines

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc} \;;\;\; \cos\beta = \frac{a^2+c^2-b^2}{2ac} \;;\;\; \cos\gamma = \frac{a^2+b^2-c^2}{2ab}$$

$$L.H.S = \frac{b^2+c^2-a^2}{2abc} + \frac{a^2+c^2-b^2}{2acb} + \frac{a^2+b^2-c^2}{2abc}$$

$$= \frac{1}{2abc}\left[b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2\right]$$

$$= \frac{a^2+b^2+c^2}{2abc} = \text{R.H.S}$$

(xv) $\quad \text{L.H.S} = 4\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{239}\right)$

$$= 2\left[2\tan^{-1}\left(\frac{1}{5}\right)\right] + \tan^{-1}\left(\frac{1}{239}\right)$$

$$= 2\left[\tan^{-1}\left(\frac{\frac{2}{5}}{1-\left(\frac{2}{5}\right)^2}\right)\right] + \tan^{-1}\left(\frac{1}{239}\right) \qquad \because 2\tan^{-1}A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$$

$$= 2\tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{1}{239}\right)$$

$$= \tan^{-1}\left(\frac{2\times\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2}\right) + \tan^{-1}\left(\frac{1}{239}\right)$$

$$= \tan^{-1}\left(\frac{120}{119}\right) + \tan^{-1}\left(\frac{1}{239}\right)$$

$$= \tan^{-1}\left(\frac{\frac{120}{119}+\frac{1}{239}}{1-\left(\frac{120}{119}\times\frac{1}{239}\right)}\right) \qquad \because \tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$= \tan^{-1}\left(\frac{28680-119}{28441+120}\right) = \tan^{-1}\left(\frac{28561}{28561}\right)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4} = R.H.S$$

(xvi) $\quad sinx + cosx = 1$

Taking square of both sides

$$\sin^2 x + \cos^2 x + 2sinxcosx = 1$$

$$1+2sinxcosx = 1$$

$$2sinxcosx = 0$$

$$\sin 2x = 0$$

$$2x = n\pi \quad \text{or} \quad x = \frac{n\pi}{2} \qquad \text{where } n \in Z$$

Solution set$= \left\{\frac{n\pi}{2}\right\}, \ n \in Z$

SECTION-C

Q3. $\quad x + 7y - z = 10$

$$4x + 8y + z = 5$$

$$2x - 3y + 2z = -5$$

$$\begin{bmatrix} 1 & 7 & -1 & 10 \\ 4 & 8 & 1 & 5 \\ 2 & -3 & 2 & -5 \end{bmatrix}$$

$$R\sim \begin{bmatrix} 1 & 7 & -1 & 10 \\ 0 & -20 & 5 & -35 \\ 0 & -17 & 4 & -25 \end{bmatrix} \qquad R_2 - 4R_1 \ ; \ R_3 - 2R_1$$

$$R\sim \begin{bmatrix} 1 & 7 & -1 & 10 \\ 0 & 1 & -\frac{1}{4} & \frac{7}{4} \\ 0 & -17 & 4 & -25 \end{bmatrix} \qquad -\frac{1}{20}R_2$$

$$R \sim \begin{bmatrix} 1 & 0 & \frac{3}{4} & -\frac{9}{4} \\ 0 & 1 & -\frac{1}{4} & \frac{7}{4} \\ 0 & 0 & -\frac{1}{4} & \frac{19}{4} \end{bmatrix} \qquad R_1 - 7R_2 \; ; \; R_3 + 17R_2$$

$$R \sim \begin{bmatrix} 1 & 0 & \frac{3}{4} & -\frac{9}{4} \\ 0 & 1 & -\frac{1}{4} & \frac{7}{4} \\ 0 & 0 & 1 & -19 \end{bmatrix} \qquad -4R_3$$

$$R \sim \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -19 \end{bmatrix} \qquad \frac{1}{4}R_3 + R_2 \; ; \; -\frac{3}{4}R_3 + R_1$$

$$x = 12, \; y = -3, \; z = -19$$

Q4. $ax^2 + bx + c = 0$, $(a \neq 0)$

(i) Let α and 3α be the roots of the equation

Sum of the roots: $\alpha + 3\alpha = -\frac{b}{a}$

$$4\alpha = -\frac{b}{a} \qquad \Rightarrow \alpha = -\frac{b}{4a}$$

Product of the roots: $(\alpha)(3\alpha) = \frac{c}{a}$

$$3\alpha^2 = \frac{c}{a}$$

$$3\left(-\frac{b}{4a}\right)^2 = \frac{c}{a} \qquad \because \alpha = -\frac{b}{4a}$$

$$\Rightarrow 3b^2 = 16ca$$

(ii) Let α and α^2 be the roots of the equation

Product of the roots: $(\alpha)(\alpha^2) = \frac{c}{a}$

$$\alpha^3 = \frac{c}{a} \qquad \Rightarrow \alpha = \left(\frac{c}{a}\right)^{\frac{1}{3}}$$

Sum of the roots: $\alpha + \alpha^2 = -\frac{b}{a}$

$$\left(\frac{c}{a}\right)^{\frac{1}{3}} + \left(\frac{c}{a}\right)^{\frac{2}{3}} = -\frac{b}{a} \qquad \because \alpha = \left(\frac{c}{a}\right)^{\frac{1}{3}}$$

(iii) Let α and $-\alpha$ be the roots of the equation, then

Product of the roots: $(\alpha)(-\alpha) = \frac{c}{a}$

$$\alpha^2 = -\frac{c}{a}$$

Sum of the roots: $\alpha + (-\alpha) = -\frac{b}{a}$

$$0 = -\frac{b}{a}$$

$$b = 0$$

(iv)   Let α and $\frac{1}{\alpha}$ be the roots of the equation.

Sum of the roots: $\alpha + \frac{1}{\alpha} = -\frac{b}{a}$

Product of the roots: $(\alpha)\left(\frac{1}{\alpha}\right) = \frac{c}{a}$

$$1 = \frac{c}{a}$$

$$c = a$$

Q5.   Let $x = \left(2^{\frac{1}{4}}\right)\left(4^{\frac{1}{8}}\right)\left(8^{\frac{1}{16}}\right)\left(16^{\frac{1}{32}}\right)\dots\infty$

Taking log on both sides

$$logx = log2^{\frac{1}{4}} + log4^{\frac{1}{8}} + log8^{\frac{1}{16}} + log16^{\frac{1}{32}} + \cdots\infty$$

$$logx = log2^{\frac{1}{4}} + log(2^2)^{\frac{1}{8}} + log(2^3)^{\frac{1}{16}} + log(2^4)^{\frac{1}{32}} + \cdots\infty$$

$$logx = \frac{1}{4}log2 + \frac{2}{8}log2 + \frac{3}{16}log2 + \frac{4}{32}log2 + \cdots\infty$$

$$logx = \left(\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \cdots\infty\right)log2 \qquad\dots\dots\dots\dots \text{(i)}$$

Let $S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \cdots\infty \qquad\dots\dots\dots\dots \text{(ii)}$

Multiplying by $\left(\frac{1}{2}\right)$

$$\frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \frac{4}{64} + \cdots\infty \qquad\dots\dots\dots\dots \text{(iii)}$$

eq. (ii) – eq. (iii)

$$\frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots\infty$$

$$\frac{1}{2}S = S_\infty$$

Where $S_\infty = \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \cdots\infty$

Here $a_1 = \frac{1}{2^2}$ and $r = \frac{1}{2}$

$\therefore$ $S_\infty = \frac{a_1}{1-r} = \frac{\frac{1}{2^2}}{\frac{1}{2}} = \frac{1}{2}$

Now $\frac{1}{2}S = \frac{1}{2}$

$$S = 1$$

Put it in eq. (i)

$$logx = (S)log2$$

$$logx = (1)log2$$

$$logx = log2$$

$$x = 2$$

Q6. $3^n + 2^{n-1} < 4^n$

Step 1:   Let the given inequality is true for $n = 2$

$$3^2 + 2^1 < 4^2$$

$$9 + 2 < 16$$

$$11 < 16 \text{ (true)}$$

Step 2:   Let the statement be true for $n = k \geq 2$

$$3^k + 2^{k-1} < 4^k \qquad \text{..........} \qquad \text{(i)}$$

Step 3:   Multiplying both sides by 4

$$4(3^k + 2^{k-1}) < 4(4^k)$$

$$4 \cdot 3^k + 4 \cdot 2^{k-1} < 4 \cdot 4^k$$

$$(3 + 1) \cdot 3^k + (2 + 2) \cdot 2^{k-1} < 4^{k+1}$$

$$3 \cdot 3^k + 3^k + 2 \cdot 2^{k-1} + 2 \cdot 2^{k-1} < 4^{k+1}$$

$$3^{k+1} + 2^k + 3^k + 2^k < 4^{k+1}$$

$$3^{\overline{k+1}} + 2^{\overline{k+1-1}} < 4^{\overline{k+1}}$$

Which is true for $n = k + 1$.

The truth for $n = k$ implies its truth for $n = k + 1$.

Hence it is true for all positive integral values of n.

Q7.

(i) $sin3\theta + sin5\theta + sin7\theta + sin9\theta = 4cos\theta sin6\theta cos2\theta$

L.H.S. $= sin3\theta + sin5\theta + sin7\theta + sin9\theta$

Using $sinA + sinB = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

L.H.S. $= 2\sin\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right) + 2\sin\left(\frac{7\theta+9\theta}{2}\right)\cos\left(\frac{7\theta-9\theta}{2}\right)$

L.H.S. $= 2\sin 4\theta \cos\theta + 2\sin 8\theta \cos\theta$

L.H.S. $= 2cos\theta(sin4\theta + sin8\theta)$

$$\text{L.H.S.} = 2cos\theta \left\{2 \sin\left(\tfrac{4\theta+8\theta}{2}\right) \cos\left(\tfrac{4\theta-8\theta}{2}\right)\right\}$$

$$\text{L.H.S.} = 4cos\theta sin6\theta cos2\theta = \text{R.H.S.}$$

(ii) $\quad cos5\theta + cos\theta + 2cos3\theta = 4cos3\theta cos^2\theta$

$$\text{L.H.S.} = cos5\theta + cos\theta + 2cos3\theta$$

Using $cosA + cosB = 2\cos\left(\tfrac{A+B}{2}\right)\cos\left(\tfrac{A-B}{2}\right)$

$$\text{L.H.S.} = 2\cos\left(\tfrac{5\theta+\theta}{2}\right)\cos\left(\tfrac{5\theta-\theta}{2}\right) + 2cos3\theta$$

$$\text{L.H.S.} = 2\cos 3\theta \cos 2\theta + 2\cos 3\theta$$

$$\text{L.H.S.} = 2cos3\theta(cos2\theta + 1)$$

$$\text{L.H.S.} = 2cos3\theta(2\cos^2\theta) \qquad\qquad \because cos2\theta + 1 = 2\cos^2\theta$$

$$\text{L.H.S.} = 4cos3\theta cos^2\theta = \text{R.H.S.}$$

Q8. Let height of the poster: $|DE| = 4feet$

Height of the man: $|AF| = 5\,feet$

Angle of Observation: $\angle D\hat{E}F = \theta = \theta_1 - \theta_2 =?$

In rt$\triangle$ EFC $\tan\theta_1 = \dfrac{7}{x}$

In rt$\triangle$ DFC $\tan\theta_2 = \dfrac{3}{x}$

$$\tan(\theta_1 - \theta_2) = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1\tan\theta_2}$$

$$\tan(\theta) = \frac{\dfrac{7}{x} - \dfrac{3}{x}}{1 + \left(\dfrac{7}{x}\right)\left(\dfrac{3}{x}\right)} = \frac{\dfrac{4}{x}}{\dfrac{x^2 + 21}{x^2}} = \frac{4x}{x^2 + 21}$$

$$\theta = \tan^{-1}\left(\frac{4x}{x^2 + 21}\right)$$