

Version No.			

ROLL NUMBER						

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

0	0	0	0	0	0	0
1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9

Answer Sheet  
No. \_\_\_\_\_

Sign. of  
Candidate \_\_\_\_\_

Sign. of  
Invigilator \_\_\_\_\_

### MATHEMATICS HSSC-I

#### SECTION – A (Marks 20)

Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

**Q.1 Fill the relevant bubble for each part. All parts carry one mark.**

1. In complex numbers, what is the multiplicative inverse of  $2i$  ?

- A.  $\frac{-i}{2}$   B.  $\frac{1}{2}$    
 C.  $\frac{-1}{2}$   D.  $\frac{i}{2}$

2. Which one of the following represents the negation of the statement  $\sim p \rightarrow q$ ?

- A.  $\sim q \rightarrow p$   B.  $\sim q \rightarrow \sim p$    
 C.  $q \rightarrow \sim p$   D.  $p \rightarrow \sim q$

3. What is the value of  $\alpha$ , if  $\begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix} = \alpha \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$ ?

- A. 3  B. 6   
 C. 9  D. 15

4. What is the solution set of an equation  $x^{-1} + \frac{1}{x^{-1}} = 2$  ?

- A.  $\{1\}$   B.  $\{-1, 1\}$    
 C.  $\{0, -1\}$   D.  $\{1, 1\}$

5. If  $-1$  is a root of  $x^3 + kx^2 - x + 2 = 0$ , then the value of  $k$  is:

- A.  $-2$   B. 2   
 C.  $-4$   D. 0

6. The partial fractions of  $\frac{7x+25}{(x+3)(x+4)}$  are:

- A.  $\frac{3}{x+4} + \frac{3}{x+3}$   B.  $\frac{3}{x+4} + \frac{4}{x+3}$    
 C.  $\frac{4}{x+4} - \frac{3}{x+3}$   D.  $\frac{-4}{x+4} + \frac{3}{x+3}$

7. For an A.P:  $2 + \frac{7}{2} + 5 + \frac{13}{2} + \dots + a_{10}$ , the value of  $S_{10}$  is:

- A.  $\frac{265}{2}$   B.  $\frac{175}{2}$    
 C.  $-\frac{95}{2}$   D. 195

8. What should be the geometric mean between  $\sqrt{2}$  and  $3\sqrt{2}$ ?
- A.  $\frac{6}{\sqrt{2}}$   B.  $6\sqrt{2}$    
 C.  $2\sqrt{2}$   D.  $\sqrt{6}$
9. For what value of  $n$ ,  ${}^n P_2 = 12$ ?
- A. 4  B. 3   
 C. 12  D. 6
10. On tossing a pair of fair coins, what is the probability of appearing both tails?
- A.  $\frac{3}{4}$   B.  $\frac{2}{4}$    
 C.  $\frac{1}{4}$   D.  $\frac{2}{3}$
11. Which one of the following is an expansion of  $(1+x)^{-1}$ ?
- A.  $1 - x + x^2 - x^3 + \dots$     
 B.  $1 + x - x^2 + x^3 + \dots$    
 C.  $1 + x + x^2 + x^3 + \dots$    
 D.  $1 - x - x^2 - x^3 + \dots$
12. How many terms are there in the expansion of  $\left[x - \frac{1}{x}\right]^{\frac{1}{2} \cdot 60}$ ?
- A. 30  B. 31    
 C. 15  D. 16
13. Which one of the following is the simplified form of  $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$ ?
- A.  $\sec\theta$   B.  $\sec^2\theta$    
 C.  $2\sec^2\theta$   D.  $2\sec\theta$
14. What will be the result if simplify  $\cos x - \left[\cos\left(x + \frac{\pi}{2}\right) + \cos\left(x - \frac{\pi}{2}\right)\right]$ ?
- A.  $\cos x$   B.  $2\cos x$    
 C. 0  D.  $(1 - \sqrt{3})\cos x$
15. If  $\theta = \frac{\pi}{2}$ , then which one of the following options is true?
- A.  $\cos 2\theta = \cos^2\theta + \sin^2\theta$    
 B.  $\sin 2\theta = 2\sin\theta \cos\theta$     
 C.  $\sin\theta = -2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$    
 D.  $\cos\theta = \cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)$
16. What is the period of a trigonometric function  $\sin\left(\frac{\pi x}{2}\right)$ ?
- A. 2  B. 4    
 C.  $\frac{1}{2}$   D.  $\frac{1}{4}$
17. In a triangle ABC, what will be e-radius opposite to vertex A?
- A.  $\frac{\Delta}{s-a}$   B.  $\frac{\Delta}{s-b}$    
 C.  $\frac{\Delta}{s-c}$   D.  $\frac{s-a}{\Delta}$
18. The value of  $\sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$  is:
- A.  $\frac{2}{\sqrt{3}}$   B.  $\frac{-2}{\sqrt{3}}$    
 C.  $\frac{1}{2}$   D.  $-\frac{1}{2}$
19. If  $\sin^{-1}x + (\sin^{-1}x + \cos^{-1}x) = \pi$ , then value of  $x$  is:
- A.  $\frac{1}{2}$   B.  $\frac{1}{\sqrt{2}}$    
 C. 0  D. -1
20. Which one of the following is a solution set of  $\sin x = \frac{1}{2}$ , where  $x \in [0, 2\pi]$ ?
- A.  $\left\{\frac{\pi}{6}, \frac{\pi}{2}\right\}$   B.  $\left\{\frac{5\pi}{6}, \frac{3\pi}{2}\right\}$    
 C.  $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$   D.  $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

**Section – B (Marks 48)**

**Q2(i):**

**Solution:**  $z_1 = 1 - 2i, z_2 = 2 + 3i, z_3 = 4 - 3i$

(a)  $\frac{\bar{z}_2}{z_3} = \frac{\overline{2+3i}}{4-3i} = \frac{2-3i}{4-3i}$

$\Rightarrow \frac{2-3i}{4-3i} \times \frac{4+3i}{4+3i} = \frac{(2-3i)(4+3i)}{(4-3i)(4+3i)} = \frac{8+6i-12i-9i^2}{16-9i^2}$

Since,  $i^2 = -1$

So,  $\frac{\bar{z}_2}{z_3} = \frac{8-6i+9}{16+9} = \frac{17}{25} - \frac{6}{25}i$

(b)  $\bar{z}_1 \cdot \bar{z}_3 = \overline{(1-2i)} \cdot \overline{(4-3i)}$

$\Rightarrow \bar{z}_1 \cdot \bar{z}_3 = (1+2i)(4+3i) = 4+3i+8i+6i^2 = 4+11i-6 \quad \because i^2 = -1$

$\Rightarrow \bar{z}_1 \cdot \bar{z}_3 = -2+11i$

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**Q2(ii):**

**Solution:**  $A \cap B = B \cap A$

We first convert it into the logical form as:  $p \wedge q = q \wedge p$

Now, we construct a truth table to prove this equality.

		L.H.S.	R.H.S.
$p$	$q$	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

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**Q2(iii):**

**Solution:** 
$$\begin{vmatrix} x & 1 & x+1 \\ 2 & x & 3 \\ x+1 & 4 & x \end{vmatrix} = 11 - 2x^2$$

To find the value of  $x$ , we expand L.H.S for first row

$$\Rightarrow x \begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix} - 1 \begin{vmatrix} 2 & 3 \\ x+1 & x \end{vmatrix} + (x+1) \begin{vmatrix} 2 & x \\ x+1 & 4 \end{vmatrix} = 11 - 2x^2$$

$$\Rightarrow x(x^2 - 12) - (2x - 3x - 3) + (x+1)(8 - x^2 - x) = 11 - 2x^2$$

$$\Rightarrow \cancel{x^3} - 12x + \cancel{1} + 3 + 8x - \cancel{x^3} - x^2 + 8 - x^2 - \cancel{x} = 11 - 2x^2$$

$$\Rightarrow -2x^2 - 12x + \cancel{11} + \cancel{2x^2} - \cancel{11} = 0$$

$$\Rightarrow -12x = 0$$

$$\Rightarrow x = 0$$

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**Q2(iv):**

**Solution:** 
$$\frac{18}{x^4} + \frac{1}{x^2} = 4$$

$$\Rightarrow \frac{18+x^2}{x^4} = 4$$

$$\Rightarrow 18 + x^2 = 4x^4$$

OR

$$\Rightarrow 4x^4 - x^2 - 18 = 0$$

$$\Rightarrow 4x^4 - 9x^2 + 8x^2 - 18 = 0$$

$$\Rightarrow x^2(4x^2 - 9) + 2(4x^2 - 9) = 0$$

$$\Rightarrow (4x^2 - 9)(x^2 + 2) = 0$$

Here,  $4x^2 - 9 = 0$  and  $x^2 + 2 = 0$

$$\Rightarrow x^2 = \frac{9}{4} \text{ and } x^2 = -2$$

Taking square-root on bothsides

$$\Rightarrow x = \pm \frac{3}{2} \text{ and } x = \pm \sqrt{2}i \text{ (Complex)}$$

Solution Set =  $\{\pm \frac{3}{2}\}$

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**Q2(v):**

**Solution:** Resolve  $\frac{3x^2+7x+28}{x(x^2+x+7)}$  into Partial Fraction

$$\Rightarrow \frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{A}{x} + \frac{Bx+C}{x^2+x+7} \quad (i)$$

Multiplying by  $x(x^2 + x + 7)$

$$\Rightarrow 3x^2 + 7x + 28 = A(x^2 + x + 7) + (Bx + C)(x) \quad (ii)$$

For  $x = 0$ , Eq. (ii) gives

$$\Rightarrow 3(0)^2 + 7(0) + 28 = A(0^2 + 0 + 7) + (B(0) + C)(0)$$

$$\Rightarrow 28 = 7A, \quad \Rightarrow A = 4$$

Expanding Eq. (ii), we get

$$\Rightarrow 3x^2 + 7x + 28 = Ax^2 + Ax + 7A + Bx^2 + Cx$$

Equating the coefficients of;

$$x^2; \quad 3 = A + B, \quad \Rightarrow 4 + B = 3, \quad \Rightarrow B = -1$$

$$x; \quad 7 = A + C, \quad \Rightarrow 4 + C = 7, \quad \Rightarrow C = 3$$

Substituting the values of  $A, B,$  and  $C$  in Eq. (i)

$$\Rightarrow \frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{4}{x} + \frac{(-1)x+3}{x^2+x+7}$$

$$\Rightarrow \frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{4}{x} - \frac{x-3}{x^2+x+7}$$

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**Q2(vi):**

**Solution:** Since,  $f(x + p) = f(x)$

Where,  $p$  is called the period of the function.

For Cosine function, we check the periodic function for  $p = 2\pi$ ,

(i.e)

$$\cos(x + 2\pi) = \cos x \cos(2\pi) - \sin x \sin(2\pi) \quad (\text{By Fundamental law of trigonometry})$$

$$\text{Here, } \cos(2\pi) = 1, \quad \text{and} \quad \sin(2\pi) = 0$$

$$\Rightarrow \cos(x + 2\pi) = \cos x (1) - \sin x (0)$$

$$\Rightarrow \cos(x + 2\pi) = \cos x$$

Hence, the period of Cosine is  $2\pi$ .

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**Q2(vii):**

**Solution:** Sum of an A.P. is;

$$S_n = \frac{n}{2} [2a_1 + (n - 1)d]$$

Sum of first 30 –terms of an A.P. is;

$$\Rightarrow S_{30} = \frac{30}{2} [2a + (30 - 1)d]$$

$$\Rightarrow S_{30} = 15[2a + 29d]$$

Similarly, the sum of first 6 –terms is;

$$\Rightarrow S_6 = 3[2a + 5d]$$

The square of sum of first 6 –terms will be;

$$\Rightarrow S_6^2 = 9(2a + 5d)^2 = 9(4a^2 + 20ad + 25d^2)$$

Since,  $S_{30} = S_6^2$

$$\Rightarrow 15(2a + 29d) = 9(4a^2 + 20ad + 25d^2)$$

$$\Rightarrow 5(2a + 29d) = 3(4a^2 + 20ad + 25d^2)$$

$$\Rightarrow 10a + 145d = 12a^2 + 60ad + 75d^2 \quad (\text{Proved})$$

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**Q2(viii):**

**Solution:**

Number of boys = 8

Number of girls = 6

Person chosen = 4

The cases for atleast one girl are;

**Case – 1:** 1 girl + 3 boys =  $\binom{6}{1} \times \binom{8}{3}$

**Case – 2:** 2 girls + 2 boys =  $\binom{6}{2} \times \binom{8}{2}$

**Case – 3:** 3 girls + 1 boy =  $\binom{6}{3} \times \binom{8}{1}$

**Case – 4:** 4 girls + 0 boy =  $\binom{6}{4} \times \binom{8}{0}$

Probability of more girls than boys is;

$$= \frac{\binom{6}{3} \times \binom{8}{1}}{\binom{14}{4}} + \frac{\binom{6}{4} \times \binom{8}{0}}{\binom{14}{4}} = \frac{160}{1001} + \frac{15}{1001} = \frac{175}{1001} = \frac{25}{143}$$

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**Q2(ix):**

**Solution:** There are 7 places to fill;

3 places of alphabet (out of 26) can be filled in =  $26 \times 25 \times 24 = 15600$

4 places of digits (out of 10 – (0 to 9)) can be filled in =  $10 \times 9 \times 8 \times 7 = 5040$

Number of different plates =  $15600 \times 5040 = 78624000$

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**Q2(x):**

**Solution:**  $\frac{1}{\sqrt{9+x}} = (9+x)^{-\frac{1}{2}} = 9^{-\frac{1}{2}} \left(1 + \frac{x}{9}\right)^{-\frac{1}{2}}$

$$\Rightarrow 3^{-1} \left(1 + \frac{x}{9}\right)^{-\frac{1}{2}} = \frac{1}{3} \left(1 + \frac{x}{9}\right)^{-\frac{1}{2}}$$

Expanding by Binomial Series

$$\Rightarrow \frac{1}{3} \left\{ 1 + \left(-\frac{1}{2}\right) \left(\frac{x}{9}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!} \left(\frac{x}{9}\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!} \left(\frac{x}{9}\right)^3 + \dots \right\}$$

$$\Rightarrow \frac{1}{3} \left\{ 1 - \frac{x}{18} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2 \cdot 1} \cdot \frac{x^2}{81} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3 \cdot 2 \cdot 1} \cdot \frac{x^3}{729} + \dots \right\}$$

$$\Rightarrow \frac{1}{3} \left\{ 1 - \frac{x}{18} + \frac{1}{8} \cdot \frac{x^2}{27} - \frac{5}{16} \cdot \frac{x^3}{729} + \dots \right\}$$

$$\Rightarrow \frac{1}{3} \left\{ 1 - \frac{x}{18} + \frac{x^2}{216} - \frac{5x^3}{11664} + \dots \right\} = \frac{1}{3} - \frac{x}{54} + \frac{x^2}{648} - \frac{5x^3}{34992} + \dots$$

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**Q2(xi):**

**Solution:** Radius = 15cm

$$\text{Interior angle} = \frac{360^\circ}{5} = 72^\circ$$

Using law of cosine, we obtain

$$\Rightarrow c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$\Rightarrow c^2 = (15)^2 + (15)^2 - 2(15)(15) \cos 72^\circ$$

$$\Rightarrow c^2 = 225 + 225 - 450(0.309)$$

$$\Rightarrow c^2 = 310.95$$

Taking square-root

$$\Rightarrow c = 17.63 \text{ cm}$$

$$\text{Perimeter of Pentagon} = 5(17.63) = 88.17 \text{ cm}$$

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**Q2(xii):**

**Solution:** 
$$\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 5\theta$$

Taking, 
$$L.H.S. = \frac{\sin 3\theta + \sin 7\theta + \sin 5\theta}{\cos 3\theta + \cos 7\theta + \cos 5\theta}$$

$$\Rightarrow L.H.S. = \frac{2 \sin\left(\frac{10\theta}{2}\right) \cos\left(\frac{4\theta}{2}\right) + \sin 5\theta}{2 \cos\left(\frac{10\theta}{2}\right) \cos\left(\frac{4\theta}{2}\right) + \cos 5\theta}$$

$$\Rightarrow L.H.S. = \frac{2 \sin 5\theta \cos 2\theta + \sin 5\theta}{2 \cos 5\theta \cos 2\theta + \cos 5\theta} = \frac{\sin 5\theta (2 \cos 2\theta + 1)}{\cos 5\theta (2 \cos 2\theta + 1)}$$

$$\Rightarrow L.H.S. = \frac{\sin 5\theta}{\cos 5\theta} = \tan 5\theta = R.H.S. \quad (\text{Proved})$$

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**Q2(xiii):**

**Solution:**  $y = \sec 2x \quad ; \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x$	$-\pi/2$	$-\pi/3$	$-\pi/6$	0	$\pi/6$	$\pi/3$	$\pi/2$
$y = \sec 2x$	-1	-2	2	1	2	-2	-1
$P(x, y)$	$\left(-\frac{\pi}{2}, -1\right)$	$\left(-\frac{\pi}{3}, -2\right)$	$\left(-\frac{\pi}{6}, 2\right)$	(0,1)	$\left(\frac{\pi}{6}, 2\right)$	$\left(\frac{\pi}{3}, -2\right)$	$\left(\frac{\pi}{2}, -1\right)$



Plot the graph on graph paper using the values calculated in the above table.

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**Q2(xiv):**

**Solution:**  $s^2 = \Delta \cot\left(\frac{\alpha}{2}\right) \cot\left(\frac{\beta}{2}\right) \cot\left(\frac{\gamma}{2}\right)$

Taking,  $R.H.S. = \Delta \cot\left(\frac{\alpha}{2}\right) \cot\left(\frac{\beta}{2}\right) \cot\left(\frac{\gamma}{2}\right)$

Where,  $\cot\left(\frac{\alpha}{2}\right) = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} ; \quad \cot\left(\frac{\beta}{2}\right) = \sqrt{\frac{s(s-b)}{(s-c)(s-a)}} ; \quad \cot\left(\frac{\gamma}{2}\right) = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$$\Rightarrow R.H.S. = \Delta \sqrt{\frac{s(s-a)}{(s-b)(s-c)} \cdot \frac{s(s-b)}{(s-c)(s-a)} \cdot \frac{s(s-c)}{(s-a)(s-b)}}$$

$$\Rightarrow R.H.S. = \Delta \sqrt{\frac{s^2 \cdot s(s-a)(s-b)(s-c)}{(s-a)^2(s-b)^2(s-c)^2}} = \Delta \sqrt{\frac{s^2 \cdot s^2}{s(s-a)(s-b)(s-c)}}$$

$$\Rightarrow R.H.S. = \Delta \left(\frac{s^2}{\Delta}\right) = s^2 = L.H.S. \quad (\text{Proved})$$

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**Q2(xv):**

**Solution:**  $\cot^{-1}\left(\frac{119}{120}\right) = 2 \sin^{-1}\left(\frac{5}{13}\right)$

Let  $y = \cot^{-1}\left(\frac{119}{120}\right)$

$$\Rightarrow \cot y = \frac{119}{120}$$

Where,  $\csc y = \sqrt{1 + \cot^2 y} = \sqrt{1 + \frac{14161}{14400}} = \sqrt{\frac{28561}{14400}}$

$$\Rightarrow \csc y = \frac{169}{120}$$

$$\Rightarrow \sin y = \frac{120}{169}$$

Here,  $\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - \frac{14400}{28561}} = \frac{119}{169}$

Using Half angle identity

$$\Rightarrow \sin\left(\frac{y}{2}\right) = \sqrt{\frac{1 - \cos y}{2}} = \sqrt{\frac{1}{2}\left(1 - \frac{119}{169}\right)} = \sqrt{\frac{25}{169}}$$

$$\Rightarrow \sin\left(\frac{y}{2}\right) = \frac{5}{13} ; \quad \Rightarrow y = 2 \sin^{-1}\left(\frac{5}{13}\right)$$

Hence,  $\cot^{-1}\left(\frac{119}{120}\right) = 2 \sin^{-1}\left(\frac{5}{13}\right)$  Proved

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**Q2(xvi):**

**Solution:** We shall show that  $2 \cos^2 \theta + 3 \cos \theta - 2 = 0$

Where,  $2 \tan^2 \theta \cos \theta = 3$

$$\Rightarrow 2 \left( \frac{\sin^2 \theta}{\cos^2 \theta} \right) \cos \theta = 3$$

$$\Rightarrow 2(1 - \cos^2 \theta) = 3 \cos \theta$$

$$\Rightarrow 2 - 2 \cos^2 \theta = 3 \cos \theta$$

$$\Rightarrow 2 \cos^2 \theta + 3 \cos \theta - 2 = 0 \quad (\text{Proved})$$

Also, to find the solution, we take  $2 \cos^2 \theta + 3 \cos \theta - 2 = 0$

$$\Rightarrow 2 \cos^2 \theta + 4 \cos \theta - \cos \theta - 2 = 0$$

$$\Rightarrow 2 \cos \theta (\cos \theta + 2) - 1(\cos \theta + 2) = 0$$

$$\Rightarrow (\cos \theta + 2)(2 \cos \theta - 1) = 0$$

Here,  $\cos \theta + 2 = 0$  and  $2 \cos \theta - 1 = 0$

$$\Rightarrow \cos \theta = -2 \text{ (Not possible)} \quad \text{and} \quad \Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3} \quad \text{and} \quad \Rightarrow \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

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**Section – C (Marks 32)**

**Q3.**

**Solution:**  $x + 2y + 3z = 3$  ;  $2x + 3y + z = 1$  ;  $3x + y + 2z = 2$

Let  $A_b$  be the augmented matrix, then

$$\Rightarrow A_b = \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 3 & 1 & 1 \\ 3 & 1 & 2 & 2 \end{array} \right]$$

To reduce  $A_b$  into reduced echelon form by elementary Row operations

$$\Rightarrow R_2 - (2)R_1, R_3 - (3)R_1 \sim^R \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & -5 & -5 \\ 0 & -5 & -7 & -7 \end{array} \right]$$

$$\Rightarrow (-1)R_2 \sim^R \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & 5 & 5 \\ 0 & -5 & -7 & -7 \end{array} \right]$$

$$\Rightarrow R_1 - (2)R_2, R_3 + (5)R_2 \sim^R \left[ \begin{array}{ccc|c} 1 & 0 & -7 & -7 \\ 0 & 1 & 5 & 5 \\ 0 & 0 & 18 & 18 \end{array} \right]$$

$$\Rightarrow \left(\frac{1}{18}\right)R_3 \sim^R \left[ \begin{array}{ccc|c} 1 & 0 & -7 & -7 \\ 0 & 1 & 5 & 5 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\Rightarrow R_1 + (7)R_3, R_2 - (5)R_3 \sim^R \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\therefore x = 0, y = 0, z = 0$$

Hence, the solution is trivial, (i.e.)  $\{(0,0,0)\}$

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**Q4.**

**Proof (a):**  $a^2 = b^2 + c^2 - 2bc \cos\alpha$

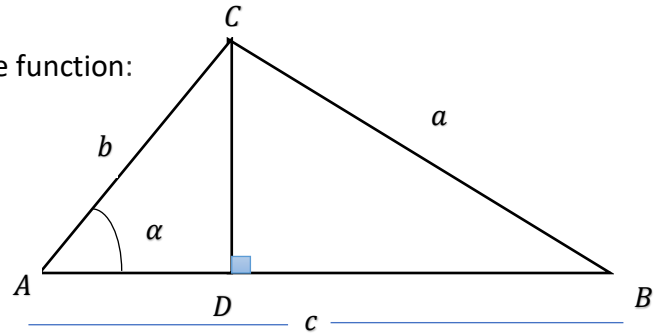
In the right triangle ACD, by the definition of cosine function:

$$\cos \alpha = \frac{AD}{b} \Rightarrow AD = b \cos\alpha \quad (i)$$

$$\Rightarrow DB = c - b \cos\alpha \quad (ii)$$

In the triangle ACD, according to Sine definition

$$\sin \alpha = \frac{CD}{b} \Rightarrow CD = b \sin\alpha \quad (iii)$$



In the triangle BDC, if we apply the Pythagorean Theorem, then

$$a^2 = (BD)^2 + (CD)^2 \quad (iv)$$

Substituting for BD and CD from equations (ii) and (iii) in equation (iv)

$$\Rightarrow a^2 = (c - b \cos\alpha)^2 + (b \sin\alpha)^2$$

$$\Rightarrow a^2 = c^2 - 2bc \cos\alpha + b^2 \cos^2 \alpha + b^2 \sin^2 \alpha$$

$$\Rightarrow a^2 = b^2(\cos^2 \alpha + \sin^2 \alpha) + c^2 - 2bc \cos\alpha$$

Where,  $\cos^2 \alpha + \sin^2 \alpha = 1$

$$\Rightarrow a^2 = b^2 + c^2 - 2bc \cos\alpha \quad (\text{Proved})$$

**Proof (b):**  $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

In right-angled triangle ADC

$$\sin\alpha = \frac{h}{b} \Rightarrow h = b \sin\alpha \quad (i)$$

In right-angled triangle BDC,

$$\sin\beta = \frac{h}{a} \Rightarrow h = a \sin\beta \quad (ii)$$

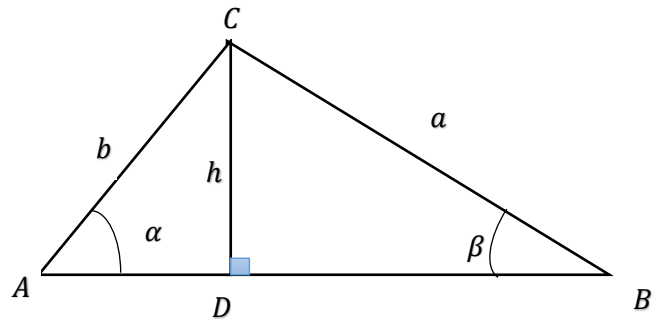
Comparing Equations (i) and (ii), we get

$$a \sin\beta = b \sin\alpha$$

$$\Rightarrow \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \quad (iii)$$

Similarly, it can also be proved that  $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$  or  $\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$

Thus,  $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$  (Proved)



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**Q5.**

**Solution:**

**(a)**  $n$ th term of the series is;

$$a_n = \frac{4}{5} \left(\frac{2}{3}\right)^{n-1}$$

For  $n = 1$

$$\begin{aligned}\Rightarrow a_1 &= \frac{4}{5} \left(\frac{2}{3}\right)^{1-1} = \frac{4}{5} \left(\frac{2}{3}\right)^0 \\ &\Rightarrow a_1 = \frac{4}{5}\end{aligned}$$

For  $n = 2$

$$\begin{aligned}\Rightarrow a_2 &= \frac{4}{5} \left(\frac{2}{3}\right)^{2-1} = \frac{4}{5} \left(\frac{2}{3}\right)^1 \\ &\Rightarrow a_2 = \frac{8}{15}\end{aligned}$$

For  $n = 3$

$$\begin{aligned}\Rightarrow a_3 &= \frac{4}{5} \left(\frac{2}{3}\right)^{3-1} = \frac{4}{5} \left(\frac{2}{3}\right)^2 \\ &\Rightarrow a_3 = \frac{16}{45}\end{aligned}$$

$\therefore \frac{4}{5} + \frac{8}{15} + \frac{16}{45} + \dots$ , the Geometric series with Common Ratio  $= \frac{2}{3}$

**(b)** To find the sum of first ten terms of the geometric series, we have

$$S_n = \frac{a_1(1-r)^n}{1-r}$$

Here,  $a_1 = \frac{4}{5}$ ,  $r = \frac{2}{3} < 1$ ,  $n = 10$

$$\Rightarrow S_{10} = \frac{\frac{4}{5} \left(1 - \frac{2}{3}\right)^{10}}{1 - \frac{2}{3}} = \frac{4}{5} \left(\frac{1}{3}\right)^{10} \times 3$$

$$\Rightarrow S_{10} = \frac{4}{5 \times 3^9} = \frac{4}{98415}$$

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**Q6.**

**Solution:** We shall prove that  $63y^2 + 84y + 19 = 0$

Where,  $y = -\frac{1}{3} + \frac{1}{3^3} + \frac{1.3}{2!} \cdot \frac{1}{3^5} + \frac{1.3.5}{3!} \cdot \frac{1}{3^7} + \dots$

$$\Rightarrow y + \frac{1}{3} = \frac{1}{3} \left( \frac{1}{3^2} + \frac{1.3}{2!} \cdot \frac{1}{3^4} + \frac{1.3.5}{3!} \cdot \frac{1}{3^6} + \dots \right)$$

$$\Rightarrow 3y + 1 + 1 = 1 + \frac{1}{3^2} + \frac{1.3}{2!} \cdot \frac{1}{3^4} + \frac{1.3.5}{3!} \cdot \frac{1}{3^6} + \dots$$

$$\Rightarrow 3y + 2 = 1 + \frac{1}{3^2} + \frac{1.3}{2!} \cdot \frac{1}{3^4} + \frac{1.3.5}{3!} \cdot \frac{1}{3^6} + \dots \quad (\text{A})$$

Let the R.H.S. of the series be identical as;

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \quad (\text{B})$$

Equating the second and third terms on R.H.S. of above equations, we get

$$nx = \frac{1}{3^2} = \frac{1}{9} \quad (\text{i})$$

$$\frac{n(n-1)}{2!}x^2 = \frac{1.3}{2!} \cdot \frac{1}{3^4} \quad (\text{ii})$$

$$\text{From Eq. (i); } x = \frac{1}{3^{2n}} \quad (\text{iii})$$

Substituting Eq. (iii) in Eq. (ii)

$$\begin{aligned} \frac{n(n-1)}{2!} \left( \frac{1}{3^{2n}} \right)^2 &= \frac{1.3}{2!} \cdot \frac{1}{3^4} \\ \Rightarrow \frac{n(n-1)}{n^2} &= \frac{1.3}{2!} \times \frac{1}{3^4} \times 2! \times 3^4 \\ \Rightarrow n-1 &= 3n \\ \Rightarrow n &= -\frac{1}{2} \end{aligned}$$

Utilizing the value of  $n$  in Eq. (iii), it gives

$$x = -\frac{2}{9}$$

Now, substituting the values of  $x$  and  $n$  in the L.H.S. of Eqs. (A) and (B), we obtain

$$3y + 2 = \left( 1 - \frac{2}{9} \right)^{-\frac{1}{2}}$$

$$\Rightarrow 3y + 2 = \left(\frac{9}{7}\right)^{\frac{1}{2}}$$

Squaring the bothsides

$$\Rightarrow (3y + 2)^2 = \frac{9}{7}$$

$$\Rightarrow 7(9y^2 + 12y + 4) = 9$$

$$\Rightarrow 63y^2 + 84y + 19 = 0$$

Hence, proved.

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**Q7.**

**Solution:**  $\cos \frac{\pi}{18} \cdot \cos \frac{\pi}{6} \cdot \cos \frac{5\pi}{18} \cdot \cos \frac{7\pi}{18} = \frac{3}{16}$

We take;

$$L.H.S. = \cos \frac{\pi}{18} \cdot \cos \frac{\pi}{6} \cdot \cos \frac{5\pi}{18} \cdot \cos \frac{7\pi}{18}$$

$$L.H.S. = \cos \frac{\pi}{6} \cdot \frac{1}{2} \left( 2 \cos \frac{5\pi}{18} \cdot \cos \frac{\pi}{18} \right) \cdot \cos \frac{7\pi}{18}$$

Where,  $2 \cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$

$$\therefore L.H.S. = \frac{\sqrt{3}}{4} \left( \cos \left( \frac{6\pi}{18} \right) + \cos \left( \frac{4\pi}{18} \right) \right) \cdot \cos \frac{7\pi}{18}$$

$$L.H.S. = \left( \frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4} \cos \left( \frac{2\pi}{9} \right) \right) \cdot \cos \frac{7\pi}{18}$$

$$L.H.S. = \frac{\sqrt{3}}{8} \cdot \cos \frac{7\pi}{18} + \frac{\sqrt{3}}{8} \left( 2 \cos \left( \frac{2\pi}{9} \right) \cdot \cos \left( \frac{7\pi}{18} \right) \right)$$

$$L.H.S. = \frac{\sqrt{3}}{8} \cdot \cos \frac{7\pi}{18} + \frac{\sqrt{3}}{8} \left( \cos \left( \frac{11\pi}{18} \right) + \cos \left( \frac{3\pi}{18} \right) \right)$$

$$L.H.S. = \frac{\sqrt{3}}{8} \cdot \cos \frac{7\pi}{18} + \frac{\sqrt{3}}{8} \cdot \cos \frac{11\pi}{18} + \frac{\sqrt{3}}{8} \cdot \frac{\sqrt{3}}{2}$$

$$L.H.S. = \frac{\sqrt{3}}{8} \left\{ \left( \cos \frac{7\pi}{18} \right) + \left( \cos \frac{11\pi}{18} \right) \right\} + \frac{3}{16}$$

$$L.H.S. = \frac{\sqrt{3}}{8} \left( 2 \cos\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{9}\right) \right) + \frac{3}{16}$$

$$L.H.S. = \frac{3}{16} = R.H.S.$$

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**Q8.**

**Solution:**  $ABC$  is a right-angled triangle with  $m\angle B = 90^\circ$ ,  $m\angle A = 30^\circ$  and  $m\overline{AB} = 3\text{cm}$

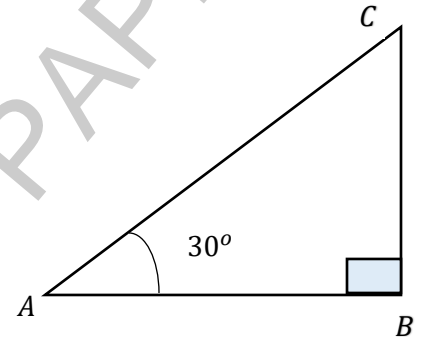
(a) First, we calculate  $m\angle C$  by

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$\Rightarrow 30^\circ + 90^\circ + m\angle C = 180^\circ$$

$$\Rightarrow m\angle C = 180^\circ - 120^\circ$$

$$\Rightarrow m\angle C = 60^\circ$$



Second,  $\cos 30^\circ = \frac{3}{m\overline{AC}} \Rightarrow m\overline{AC} = 2\sqrt{3}\text{cm}$

Third,  $\sin 30^\circ = \frac{m\overline{BC}}{2\sqrt{3}} \Rightarrow m\overline{BC} = \sqrt{3}\text{cm}$

(b) Area of triangle ( $\Delta ABC$ ) =  $\frac{1}{2}(m\overline{AB})(m\overline{BC})$

$$\Rightarrow \Delta = \frac{1}{2}(3)(\sqrt{3})$$

$$\Rightarrow \Delta = \frac{3\sqrt{3}}{2} \text{ cm}^2$$

(c) Radius of circum-circle ( $R$ ) =  $\frac{abc}{4\Delta}$

$$\Rightarrow R = \frac{(3)(2\sqrt{3})(\sqrt{3})}{4\left(\frac{3\sqrt{3}}{2}\right)} = \sqrt{3}\text{cm}$$

(d) Radius of in-circle ( $r$ ) =  $\frac{\Delta}{s}$

Where,  $s = \frac{a+b+c}{2} = \frac{3+2\sqrt{3}+\sqrt{3}}{2} = \frac{3(1+\sqrt{3})}{2}$



$$\Rightarrow r = \frac{\left(\frac{3\sqrt{3}}{2}\right)}{\left(\frac{3(1+\sqrt{3})}{2}\right)}$$

$$\Rightarrow r = \frac{\sqrt{3}}{1+\sqrt{3}}$$

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