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Answer Sheet No. _____

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MATHEMATICS SSC-II (3rd Set)

(Science Group) (Curriculum 2006)

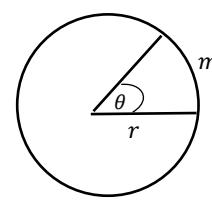
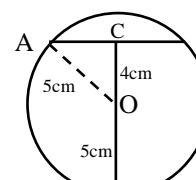
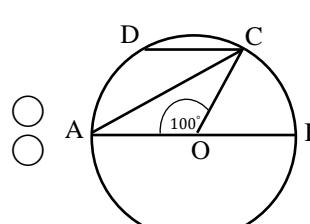
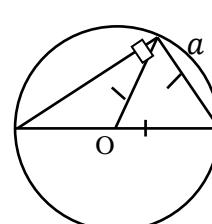
SECTION – A (Marks 15)

Time allowed: 20 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

Q.1 Fill the relevant bubble for each part. All parts carry one mark.

- (1) Cancellation of x on both sides of $6x^2 = 21x$ means:
- A. The loss of one root B. No loss of any root
 C. The gain of one root D. Undefined solution
- (2) If $b^2 - 4ac > 0$ is a perfect square then roots of $ax^2 + bx + c = 0$ are?
- A. Irrational, Equal B. Rational, Equal
 C. Rational, Unequal D. Irrational, Unequal
- (3) On simplifying $(7 + 5\omega + 5\omega^2)^2$ we get:
- A. 4 B. 12
 C. 17 D. 144
- (4) If y^2 varies inversely as x^3 then:
- A. $y^2 = k x^3$ B. $y^2 = \frac{k}{x^3}$
 C. $\frac{y^2}{x^3} = 1$ D. $y^2 x^3 = 1$
- (5) Partial fractions of $\frac{x^2+1}{(x+1)(x-1)}$ are of the form:
- A. $1 - \frac{A}{(x+1)} + \frac{B}{(x-1)}$ B. $1 + \frac{A}{(x+1)} + \frac{B}{(x-1)}$
 C. $1 + \frac{A}{(x+1)} + \frac{Bx}{(x-1)}$ D. $\frac{A}{(x+1)} + \frac{B}{(x-1)}$
- (6) If $A \cap B = \emptyset$, then set A and B are:
- A. Subsets of each other B. Overlapping sets
 C. Disjoint sets D. Equal sets

- (7) If $f : A \rightarrow B$ and range of $f \neq B$ then f is a/an:
- A. Into function B. Onto function
 C. Bijective function D. Injective function
- (8) If $Y = X + 5$ then $\bar{Y} = ?$
- A. \bar{X} B. 5
 C. $\bar{X} + 5$ D. $5\bar{X}$
- (9) $\sum(X - \bar{X}) = ?$
- A. 2 B. 1
 C. -1 D. 0
- (10) In which of the following quadrants θ lies when $\sin \theta < 0, \sec \theta < 0$?
- A. I B. II
 C. III D. IV
- (11) $\sec \theta \cot \theta = ?$
- A. cosec θ B. $\tan \theta$
 C. $\sin \theta$ D. $\cos \theta$
- (12) What is the value of m , if $r = 15$ and $\theta = \frac{\pi}{3}$?
- A. 5π B. $\frac{\pi}{5}$
 C. 45π D. $\frac{45}{\pi}$
- 
- (13) What is the length of chord intercepted at 4cm away from the centre of the circle?
- A. 4cm B. 6cm
 C. 7cm D. 9cm
- 
- (14) If $\overline{DC} \parallel \overline{AB}$ and $\angle AOC = 100^\circ$ (in the figure), then $\angle ACD = ?$
- A. 30° B. 40°
 C. 50° D. 60°
- 
- (15) In the adjoining figure, length of the escribed radii a is:
- A. a B. $2a$
 C. $3a$ D. $\frac{1}{2}a$
- 

SOLUTION QUESTION MODEL PAPER (3rd Set) SSC-II
MATHEMATICS

SECTION-A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	B	A	B	B	C	A	C	C	A	A	D	B	C	B

SECTION-B

Question 2

$$(i) \quad \frac{2x+1}{x+2} - \frac{2x+4}{2x+8} = 0$$

$$(2x+1)(2x+8) - (2x+4)(x+2) = 0$$

$$4x^2 + 16x + 2x + 8 - 2x^2 - 4x - 4x - 8 = 0$$

$$x^2 + 5x + 0 = 0$$

Applying the quadratic formula for x

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Where } a = 1, b = 5, c = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 0}}{2}$$

$$x = \frac{-5 \pm 5}{2}$$

$$x = -5 \text{ or } x = 0$$

Solution Set = $\{-5, 0\}$

$$(ii) \quad 3 \cdot 3^{2x+1} - 10 \cdot 3^x + 1 = 0$$

$$3 \cdot 3^{2x} - 10 \cdot 3^x + 1 = 0$$

$$9(3^x)^2 - 10(3^x) + 1 = 0$$

Let $3^x = y \rightarrow \text{eqn - I}$

$$9y^2 - 10y + 1 = 0$$

$$9y^2 - 9y - y + 1 = 0$$

$$9y(y - 1) - 1(y - 1) = 0$$

$$(y - 1)(9y - 1) = 0$$

$$y - 1 = 0 \quad \text{or} \quad 9y - 1 = 0$$

$$y = 1 \quad \text{or} \quad y = \frac{1}{9}$$

Putting the value of y in eqn - I

$$3^x = 1 \quad \text{or} \quad 3^x = \frac{1}{9}$$

$$3^x = 3^0 \quad \text{or} \quad 3^x = 3^{-2}$$

$$x = 0 \quad \text{or} \quad x = -2$$

(iii) If θ, \emptyset are the roots of $y^2 - 7y + 9 = 0$, then

$$\text{Sum of the roots} = \theta + \emptyset = -\frac{\text{coeff of } y}{\text{coeff of } y^2} = -\frac{-7}{1} = 7$$

$$\text{Product of the roots} = \theta \emptyset = \frac{\text{constt term}}{\text{coeff of } y^2} = \frac{9}{1} = 9$$

If roots of the required equation are $2\theta, 2\emptyset$ then,

$$\text{Sum of the roots: } S = 2\theta + 2\emptyset = 2(\theta + \emptyset) = 2(7) = 14$$

$$\text{Product of the roots: } P = (2\theta)(2\emptyset) = 4(\theta\emptyset) = 4(9) = 36$$

$$\text{Required quadratic equation: } y^2 + Sy + P = 0$$

$$y^2 + 14y + 36 = 0$$

(iv) If $x\text{cm}$ be the breadth of the rectangle, then it's length $= (x + 5)\text{cm}$

$$\text{Area of the rectangle: } x(x + 5) = 50$$

$$x^2 + 5x - 50 = 0$$

$$(x - 5)(x + 10) = 0$$

$$x - 5 = 0 \text{ or } x + 10 = 0$$

$$x = 5 \text{ or } x = -10$$

$$\text{Breadth: } x = 5\text{cm} \quad (\text{neglecting the negative value})$$

$$\text{Length: } x + 5 = 5 + 5 = 10\text{cm}$$

(v) If a be the fourth proportional, then

$$(x^3 - y^3) : (x^2 - y^2) :: (y^2 + 2xy + y^2) : a$$

Product of Extremes = Product of Means

$$(a)(x^3 - y^3) = (x^2 - y^2)(y^2 + 2xy + y^2)$$

$$a = \frac{(x^2 - y^2)(2y^2 + 2xy)}{(x^3 - y^3)}$$

$$a = \frac{(x - y)(x + y)(x + y)2y}{(x - y)(x^2 + xy + y^2)}$$

$$a = \frac{2y(x + y)^2}{(x^2 + xy + y^2)}$$

(vi) $I \propto E$ and $I \propto \frac{1}{R}$ $\Rightarrow I \propto \frac{E}{R}$ $\Rightarrow I = \frac{kE}{R}$ $\rightarrow \text{eqn} - I$

For $I = 32 \text{ amp}$, $E = 1280 \text{ volts}$ and $R = 80 \text{ ohm}$

$$32 = \frac{k(1280)}{80}$$

$$k = \frac{(80)(32)}{1280} = 2$$

Putting k 's value in $\text{eqn} - I$

$$I = \frac{2E}{R}$$

When $E = 1500 \text{ volts}$ and $R = 180 \text{ ohm}$

$$I = \frac{2(1500)}{180} = \frac{50}{3} \text{ amp}$$

$$(vii) \quad \frac{4x+2}{2(x-1)(x^2+1)^2} = \frac{2(2x+1)}{2(x-1)(x^2+1)^2}$$

$$\frac{(2x+1)}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} \quad \rightarrow eqn - I$$

Multiplying both sides by $(x-1)(x^2+1)^2$

$$2x+1 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1) \quad \rightarrow eqn - II$$

For A put $x-1=0$ or $x=1$ in $eqn - I$

$$2(1)+1 = A(1^2+1)^2 + 0 + 0 \quad \Rightarrow A = \frac{3}{4}$$

Simplifying $eqn - II$

$$2x+1 = A(x^4+2x^2+1) + B(x^4-x^3+x^2-x) + C(x^3-x^2+x-1) + D(x^2-x) + E(x-1)$$

Equating the coefficients of like powers of x^4, x^3, x^2, x

$$\text{Coeff of } x^4: \quad 0 = A + B \quad \Rightarrow B = -A = -\frac{3}{4}$$

$$\text{Coeff of } x^3: \quad 0 = -B + C \quad \Rightarrow C = B = -\frac{3}{4}$$

$$\text{Coeff of } x^2: \quad 0 = 2A + B - C + D \quad \Rightarrow D = C - B - 2A = -\frac{3}{4} + \frac{3}{4} - 2\left(\frac{3}{4}\right) = -\frac{3}{2}$$

$$\text{Coeff of } x: \quad 2 = -B + C - D + E \quad \Rightarrow E = D - C + B + 2 = -\frac{3}{2} + \frac{3}{4} - \frac{3}{4} + 2 = \frac{1}{2}$$

Substituting the values of A, B, C, D and E in $eqn - I$

$$\frac{2x+1}{(x-1)(x^2+1)} = \frac{3}{4(x-1)} - \frac{3(x+1)}{4(x^2+1)} - \frac{(3x-1)}{2(x^2+1)^2}$$

$$(viii) \quad U = \{1, 2, 3, \dots, 20\}, \quad A = \{1, 2, 3, \dots, 10\}, \quad B = \{2, 4, 6, 8, 10, 12, 14, 16\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16\}, \quad A \cap B = \{2, 4, 6, 8, 10\}$$

$$(A \cap B)^c = U - (A \cap B) = \{1, 3, 5, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

$$(A \cup B) - (A \cap B)^c = \{1, 2, 3, \dots, 10, 12, 14, 16\} - \{1, 3, 5, 7, 9, 11, 12, 13, 14, \dots, 19, 20\} = \{2, 4, 6, 8, 10\}$$

(ix)

x	90	80	70	90	$\Sigma x = 330$
x^2	8100	6400	4900	8100	$\Sigma x^2 = 27500$

Number of values: $n = 4$

$$\text{Variance: } S^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = \frac{27500}{4} - \left(\frac{330}{4}\right)^2 = 68.75$$

$$\text{Standard Deviation: } S = \sqrt{68.75} = 8.29$$

(x) Consider a right triangle ABC with $m\angle C = 90^\circ$

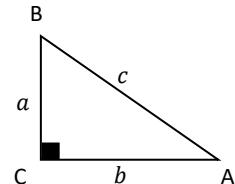
$$\tan \theta = \frac{a}{b} = \frac{\sqrt{7}}{2} \quad \Rightarrow a = \sqrt{7}, b = 2$$

By Pythagoras Theorem

$$c^2 = a^2 + b^2 \quad c = \sqrt{(\sqrt{7})^2 + (2)^2} = \sqrt{11}$$

$$\sin \theta = \frac{a}{c} = \frac{\sqrt{7}}{\sqrt{11}} \quad \cos \theta = \frac{b}{c} = \frac{2}{\sqrt{11}} \quad \tan \theta = \frac{a}{b} = \frac{\sqrt{7}}{2}$$

$$\csc \theta = \frac{c}{a} = \frac{\sqrt{11}}{\sqrt{7}} \quad \sec \theta = \frac{\sqrt{11}}{2} \quad \cot \theta = \frac{b}{a} = \frac{2}{\sqrt{7}}$$



(xi)

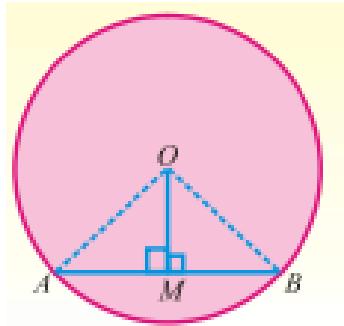
Figure:

Given: \overline{AB} is the chord of a circle with centre at O so that $\overline{OM} \perp$ chord \overline{AB} .

To prove: M is the midpoint of chord \overline{AB} i.e. $m\overline{AM} = m\overline{BM}$.

Construction: Join O to A and B.

Proof:



Statements	Reasons
In $\angle rt \Delta^s OAM \leftrightarrow OBM$ $m\angle OMA = m\angle OMB = 90^\circ$ $Hyp m\overline{OA} = Hyp m\overline{OB}$ $m\overline{OM} = m\overline{OM}$ $\therefore \Delta OAM \cong \Delta OBM$ Hence, $m\overline{AB} = m\overline{BM}$ $\Rightarrow \overline{OM}$ bisects the chord \overline{AB} .	Given Radii of the same circle Common In $\angle rt \Delta^s H.S \cong H.S$ Corresponding sides of congruent triangles.

(xii)

In the figure given that

$$m\overline{CE} = m\overline{DE} = 2\text{cm}, \quad m\overline{OA} = m\overline{OB} = m\overline{OE} = 3\text{cm}$$

$$m\overline{PA} = m\overline{BP} = 8\text{ cm}$$

In an Isosceles ΔPCD

$$m\overline{PC} = m\overline{PD} \quad m\overline{PC} + m\overline{PD} = ?$$

In $\text{rt.}\Delta POA$, (Pythagoras Theorem)

$$(m\overline{OP})^2 = (m\overline{OA})^2 + (m\overline{AP})^2 = 3^2 + 8^2 = 73$$

$$m\overline{OP} = \sqrt{73} = 8.54$$

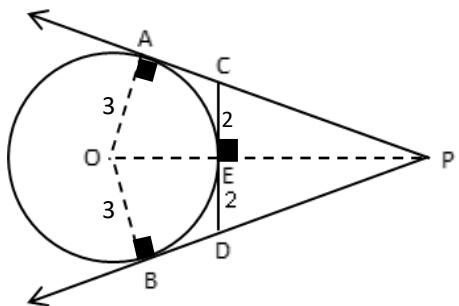
$$\overline{PE} = \overline{OP} - \overline{OE} = 8.54 - 3 = 5.54$$

In $\text{rt.}\Delta PCE$, (Pythagoras Theorem)

$$(m\overline{PC})^2 = (m\overline{CE})^2 + (m\overline{PE})^2 = 2^2 + 5.54^2 = 34.69$$

$$m\overline{PC} = \sqrt{34.69} = 5.89\text{cm} = m\overline{PD}$$

$$m\overline{PC} + m\overline{PD} = 5.89 + 5.89 = 11.78\text{cm}$$



(xiii)

Given in $\triangle ABC$, $m\angle CAD = a = 30^\circ$, $m\angle ACB = d = 45^\circ$

O is the center of the circle

from figure \widehat{ADC} is a semi-circle

$$m\angle ADC = f = 90^\circ$$

$$e = 180 - (a + f) = 180 - (30 + 90) = 60^\circ$$

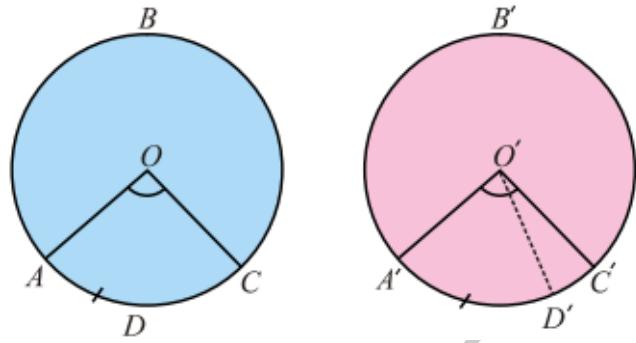
Again from fig, \widehat{ABC} is a semi-circle

$$m\angle ABC = c = 90^\circ$$

$$b = 180^\circ - (d + c) = 180^\circ - (45^\circ + 90^\circ) = 45^\circ$$

(xiv)

Figure:



Given: ABC and A'B'C' are two congruent circles with centers O and O' respectively so that $\overline{AC} = \overline{A'C'}$

To prove: $\angle AOC \cong \angle A'O'C'$

Construction: Let if possible $m\angle AOC \neq m\angle A'O'C'$ then consider $\angle AOC \cong \angle A'O'D'$

Proof:

Statements	Reasons
$\angle AOC = \angle A'O'D'$	Construction
$\therefore \widehat{AC} \cong \widehat{A'D'} \quad \text{eqn - I}$	Areas subtended by equal central angles in congruent circles
$m\overline{AC} = m\overline{A'D'} \quad \text{eqn - II}$	If two arcs of a circle are congruent then corresponding chords are equal
But $m\overline{AC} = m\overline{A'C'} \quad \text{eqn - III}$	Given
$\therefore m\overline{A'C'} = m\overline{A'D'} \quad \text{Using eqns - II & III}$	
Which is only possible, if C' coincides with D'.	
Hence $m\angle A'O'C' = m\angle A'O'D' \quad \text{eqn - IV}$	
But $m\angle AOC = m\angle A'O'D' \quad \text{eqn - V}$	Construction
$\Rightarrow \angle AOC = \angle A'O'C'$	Using eqns - IV & V

SECTION – C

Q3. $x^4 - 4x^3 - 3x^2 + 4x + 1 = 0$

$$x^2 - 4x - 3 + \frac{4}{x} + \frac{1}{x^2} = 0 \quad (\text{Divided by } x^2)$$

$$x^2 + \frac{1}{x^2} - 4x + \frac{4}{x} - 3 = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 4\left(x - \frac{1}{x}\right) - 3 = 0$$

$$\text{Let } x - \frac{1}{x} = y \implies \left(x^2 + \frac{1}{x^2}\right) = y^2 + 2$$

$$(y^2 + 2) - 4y - 3 = 0$$

$$y^2 - 4y - 1 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ with } a = 1, b = -4, c = -1$$

$$y = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$

$$y = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$$x - \frac{1}{x} = 2 \pm \sqrt{5} \quad \Rightarrow x^2 - (2 \pm \sqrt{5})x - 1 = 0$$

$$x^2 - (2 + \sqrt{5})x - 1 = 0 \quad ; \quad x^2 - (2 - \sqrt{5})x - 1 = 0$$

$$x = \frac{(2 + \sqrt{5}) + \sqrt{(2 + \sqrt{5})^2 - 4(1)(-1)}}{2(1)} \quad ; \quad x = \frac{(2 - \sqrt{5}) - \sqrt{(2 - \sqrt{5})^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{(2 + \sqrt{5}) + \sqrt{13 + 4\sqrt{5}}}{2} \quad ; \quad x = \frac{(2 - \sqrt{5}) - \sqrt{13 - 4\sqrt{5}}}{2}$$

$$\text{Solution set} = \left\{ \frac{(2+\sqrt{5})+\sqrt{13+4\sqrt{5}}}{2}, \frac{(2-\sqrt{5})-\sqrt{13-4\sqrt{5}}}{2} \right\}$$

Q4 $U = \{5, 6, 7, 8, 9, \dots, 20\}, A = \{6, 8, 10, \dots, 20\}, B = \{5, 7, 11, 13, 17, 19\}$

(i) $A \cup B = \{5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20\}$
 $(A \cup B)^c = U - (A \cup B) = \{9, 15\} \rightarrow \text{eqn - I}$
 $A^c = U - A = \{5, 7, 9, 11, 13, 15, 17, 19\}, \quad B^c = U - B = \{6, 8, 9, 10, 12, 14, 15, 16, 18\}$
 $A^c \cap B^c = \{9, 15\} \rightarrow \text{eqn - II}$
From eqns - I & II
 $(A \cup B)^c = A^c \cap B^c$

(ii) $A \cap B = \emptyset$
 $(A \cap B)^c = U - A \cap B = \{5, 6, 7, 8, 9, \dots, 20\} \rightarrow \text{eqn - III}$
 $A^c \cup B^c = \{5, 6, 7, 8, 9, \dots, 20\} \rightarrow \text{eqn - IV}$
From eqns - III & IV
 $(A \cap B)^c = A^c \cup B^c$

Q5.

Class Interval	Mid Value (x)	f	log x	f log x	f/x
0 — 10	05	3	0.6989	2.0969	0.6
10 — 20	15	4	1.17609	4.7043	0.266
20 — 30	25	5	1.3979	6.9897	0.2
30 — 40	35	6	1.54406	9.26440	0.1714
40 — 50	45	7	1.65321	11.57240	0.1555
		$\Sigma = 25$		$\Sigma = 34.62778$	$\Sigma = 1.3929$

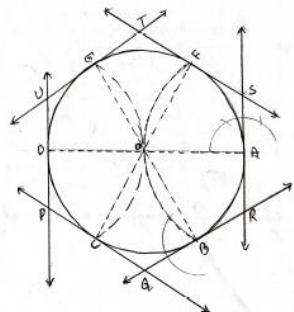
$$\text{Geometric Mean} = \text{Antilog} \left[\frac{\sum(f \log x)}{\sum f} \right] = \text{Antilog} \left(\frac{34.62778}{25} \right) = 24.273$$

$$\text{Harmonic Mean} = \frac{\sum f}{\sum \left(\frac{f}{x} \right)} = \frac{25}{1.3929} = 17.95$$

Q 6. Construction Steps:

- Draw a circle of radius 5 cm.
- Draw diameter \overline{AD}
- From point A draw an arc of radius \overline{AO} , which cuts the circle at points B and F.
- Join B with O and extend it to meet the circle at E.
- Join F with O and extend it to meet the circle at C.
- Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T, U respectively.

Thus, $PQRSTU$ is the circumscribed regular hexagon (figure)



Q7

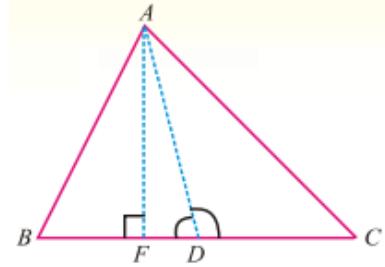
Figure:

Given: In ΔABC , the median \overline{AD} bisects \overline{BC} .

i.e. $m\overline{BD} = m\overline{CD}$.

To prove: $(AB)^2 + (AC)^2 = 2(BD)^2 + 2(AD)^2$

Construction: Draw $\overline{AF} \perp \overline{BC}$



Proof:

(04)

Statements	Reasons
In ΔADB $\angle ADB$ is acute at D	
$\therefore (AB)^2 = (BD)^2 + (AD)^2 - 2 m\overline{BD} \cdot m\overline{FD}$ (i)	In any triangle, the square on the side opposite to acute angle is equal to the sum of squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and projection on it of the other.
In ΔADC $\angle ADC$ is obtuse at D	
$\therefore (AC)^2 = (CD)^2 + (AD)^2 + 2 m\overline{CD} \cdot m\overline{FD}$	In an obtuse angled triangle, the square on the side opposite to obtuse angle is equal to the sum of squares on the sides containing that obtuse angle diminished by twice the rectangle contained by one of those sides and projection on it of the other.
$(AC)^2 = (BD)^2 + (AD)^2 + 2 m\overline{BD} \cdot m\overline{FD}$ (ii) $(AB)^2 + (AC)^2 = 2(BD)^2 + 2(AD)^2$	$(BD)^2 = (CD)^2$ Adding (i) and (ii)

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