Exercise 1.6

Q1. Use matrices, if possible, to solve the following systems of linear equations by:

(i) the matrix inversion method

(ii) the Cramer's rule

(i) $2x - 2y = 4$, $3x + 2y = 6$

(ii) $2x + y = 3$, $6x + 5y = 1$

(iii) $4x + 2y = 8$, $3x - y = -1$

(iv) $3x - 2y = -6$, $5x - 2y = -10$

(v) $3x - 2y = 4$, $-6x + 4y = 7$

(vi) $4x + y = 9$, $-3x - y = -5$

(vii) $2x - 2y = 4$, $-5x - 2y = -10$

(viii) $3x - 4y = 4$, c

(i) Solution by Matrix Inversion Method:

(i) $2x - 2y = 4$

$3x + 2y = 6$

Step 1

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Step 2

The coefficient matrix $M = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$ is non-singular because

$$\det M = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = 2 \times 2 - 3 \times (-2) = 4 + 6 = 10 \neq 0$$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|}\, Adj\, M \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2\times 4 + 2\times 6 \\ -3\times 4 + 2\times 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 8 + 12 \\ -12 + 12 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $x = 2$, $y = 0$

(ii) **2x + y = 3**

6x + 5y = 1

Step 1

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = 2 \times 5 - 1 \times 6 = 10 - 6 = 4 \neq 0$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|}\, Adj\, M \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 5\times 3+(-1)\times 1 \\ -6\times 3+2\times 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$\Rightarrow \quad x = \frac{7}{2}, \; y = -4$

(iii) 4x + 2y = 8

3x - y = - 1

Step 1

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = 4 \times (-1) - 2 \times 3 = -4 - 6 = -10 \neq 0$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|}\, Adj\; M \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$= \frac{1}{-10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$= \frac{1}{-10}\begin{bmatrix} (-1)\times 8+(-2)\times(-1) \\ -3\times 8+4\times(-1) \end{bmatrix}$$

$$= \frac{1}{-10}\begin{bmatrix} -8+2 \\ -24-4 \end{bmatrix}$$

$$= \frac{1}{-10}\begin{bmatrix} -6 \\ -28 \end{bmatrix} \qquad = \begin{bmatrix} \frac{3}{5} \\ \frac{14}{5} \end{bmatrix}$$

$\Rightarrow \quad x = \frac{3}{5}, \; y = \frac{14}{5}$

(iv) 3x – 2y = - 6

5x - 2y = - 10

Step 1

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix} = 3 \times (-2) - 5 \times (-2) = -6 + 10 = 4 \neq 0$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\, M \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} (-2) \times (-6) + (-2) \times (-10) \\ (-5) \times (-6) + 3 \times (-10) \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -12 - 20 \\ 30 - 30 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -8 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad x = -2, \; y = 0$

(v) 3x – 2y = 4

-6x + 4y = 7

Step 1

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$ is singular because

det M = $\begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix} = 3 \times 4 - (-6) \times (-2) = 12 - 12 = 0$

So, M is a singular matrix. Hence the system of linear equations has no solution

(vi) **4x + y = 9**

-3x - y = - 5

Step 1

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix} = 4 \times (-1) - -3 \times 1 = -4 + 3 = -1 \neq 0$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\, M \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$= \frac{1}{-1} \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$= \frac{1}{-1} \begin{bmatrix} (-1) \times 9 + (-1) \times (-5) \\ 3 \times 9 + 4 \times (-5) \end{bmatrix}$$

$$= -1 \begin{bmatrix} -9 + 5 \\ 27 - 20 \end{bmatrix}$$

$$= -1 \begin{bmatrix} -4 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$$

$\Rightarrow \quad x = 4\ ,\ y = -7$

(vii) 2x – 2y = 4

-5x - 2y = - 10

Step 1

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

Step 2

The coefficient matrix M = $\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix} = 2\ \times(-2) - 5\ \times 2$ = - 4 - 10 = -14 $\neq 0$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|} Adj\ M \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$= \frac{1}{-14} \begin{bmatrix} -2 & -2 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$= \frac{1}{-14} \begin{bmatrix} (-2) \times 4 + 2 \times (-10) \\ 5 \times 4 + 2 \times (-10) \end{bmatrix}$$

$$= \frac{1}{-14} \begin{bmatrix} -8 - 20 \\ 20 - 20 \end{bmatrix}$$

$$= \frac{1}{-14} \begin{bmatrix} -28 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad x = 2\ ,\ y = 0$

(viii) 3x – 4y = 4

x + 2y = 8

Step 1

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Step 2

The coefficient matrix M $=\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$ is non-singular because

det M $= \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} = 3 \times 2 - (-4) \times 1 = 6 + 4 = 10 \neq 0$

Step 3

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|M|}\, Adj\ M \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2 \times 4 + 4 \times 8 \\ (-1) \times 4 + 3 \times 8 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 8 + 32 \\ -4 + 24 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 40 \\ 20 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$\Rightarrow \quad x = 4$, y = 2

(ii) Solution by Cramer's Rule:

(i) 2x – 2y = 4

3x + 2y = 6

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = 2 \times 2 - 3 \times (-2)$

$= 4 + 6 = 10 \neq 0$

$A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$

$|A_x| = \begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix} = 4 \times 2 - 6 \times (-2)$

$= 8 + 12 = 20 \neq 0$

$A_y = \begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} = 2 \times 6 - 3 \times 4$

$= 12 - 12 = 0 \neq 0$

$x = \frac{|A_x|}{|A|} = \frac{20}{10} = 2$

$y = \frac{|A_y|}{|A|} = \frac{0}{10} = 0$

So, $x = 2$ **and** $y = 0$

(ii) **2x + y = 3**

6x + 5y = 1

$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$

$|A| = \left|\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\right| = 2 \times 5 - 1 \times 6$

$= 10 - 6 = 4 \neq 0$

$A_x = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$

$|A_x| = \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} = 3 \times 5 - 1 \times 1$

$= 15 - 1 = 14 \neq 0$

$A_y = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix} = 2 \times 1 - 3 \times 6$

$= 2 - 18 = -16 \neq 0$

$x = \frac{|A_x|}{|A|} = \frac{14}{4} = \frac{7}{2}$

$y = \frac{|A_y|}{|A|} = \frac{-16}{4} = -4$

So, $x = \frac{7}{2}$ **and** $y = -4$

(iii) **4x + 2y = 8**

3x - y = - 1

$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = 4 \times (-1) - 3 \times 2$

$= -4 - 6 = -10 \neq 0$

$A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$

$|A_x| = \begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix} = 8 \times (-1) - 2 \times (-1)$

$= -8 + 2 = -6 \neq 0$

$A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix} = 4 \times (-1) - 3 \times 8$

$= -4 - 24 = -28 \neq 0$

$x = \frac{|A_x|}{|A|} = \frac{-6}{-10} = \frac{3}{5}$

$y = \frac{|A_y|}{|A|} = \frac{-28}{10} = \frac{14}{5}$

So, $x = \frac{3}{5}$ and $y = \frac{14}{5}$

(iv) **$3x - 2y = -6$**

$5x - 2y = -10$

$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$

$A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix} = 3 \times (-2) - 5 \times (-2)$

$= -6 + 10 = 4 \neq 0$

$A_x = \begin{bmatrix} -6 & -2 \\ -10 & -2 \end{bmatrix}$

$|A_x| = \begin{vmatrix} -6 & -2 \\ -10 & -2 \end{vmatrix}$

$= -6 \times (-2) - (-10) \times (-2)$

$= 12 - 20 = -8$

$A_y = \begin{bmatrix} 3 & -6 \\ 5 & -10 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 3 & -6 \\ 5 & -10 \end{vmatrix}$

$= \quad 3 \times (-10) - 5 \times (-6)$

$= \quad -30 + 30 \quad = \quad 0$

$x \quad = \quad \frac{|A_x|}{|A|} \quad = \quad \frac{-8}{4} \quad = \quad -2$

$y \quad = \quad \frac{|A_y|}{|A|} \quad = \quad \frac{0}{4} \quad = \quad 0$

So, $\boldsymbol{x = -2}$ **and** $\boldsymbol{y = 0}$

(v) 3x – 2y = 4

-6x + 4y = 7

$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

$A \quad = \quad \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$

$|A| \quad = \quad \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix} \quad = \quad 3 \times 4 - (-6) \times (-2)$

$= \quad 12 - 12 \quad = \quad 0$

Since it is a singular matrix therefore solution is not possible.

(vi) 4x + y = 9

-3x - y = - 5

$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$A \quad = \quad \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$

$|A| \quad = \quad \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix} \quad = \quad 4 \times (-1) - (-3) \times 1$

$= \quad -4+3 \quad = \quad -1 \quad \neq \quad 0$

$A_x \quad = \quad \begin{bmatrix} 9 & 1 \\ -5 & -1 \end{bmatrix}$

$|A_x| \quad = \quad \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix} \quad = \quad 9 \times (-1) - (-5) \times 1$

$= \quad -9+5 \quad = \quad -4 \quad \neq \quad 0$

$A_y \quad = \quad \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$

$|A_y| \quad = \quad \begin{vmatrix} 4 & 9 \\ -3 & -5 \end{vmatrix} \quad = \quad 4 \times (-5) - (-3) \times 9$

$= \quad -20+27 \quad = \quad 7 \quad \neq \quad 0$

$x \quad = \quad \frac{|A_x|}{|A|} \quad = \quad \frac{-4}{-1} \quad = \quad 4$

$y \quad = \quad \frac{|A_y|}{|A|} \quad = \quad \frac{7}{-1} \quad = \quad -7$

So, $x = 4$ **and** $y = -7$

(vii) **2x – 2y = 4**

-5x - 2y = - 10

$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$

$A \quad = \quad \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$

$|A| \quad = \quad \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix} \quad = \quad 2 \times (-2) - (-5) \times (-2)$

$= \quad -4-10 \quad = \quad -14 \quad \neq \quad 0$

$A_x \quad = \quad \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$

$|A_x| \quad = \quad \begin{vmatrix} 4 & -2 \\ -10 & -2 \end{vmatrix} \quad = \quad 4 \times (-2) - (-10) \times 2$

$= -8 - 20 = -28$

$A_y = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 2 & 4 \\ -5 & -10 \end{vmatrix} = 2 \times (-10) - (-5) \times 4$

$= -20 + 20 = 0$

$x = \frac{|A_x|}{|A|} = \frac{-28}{-14} = 2$

$y = \frac{|A_y|}{|A|} = \frac{0}{-14} = 0$

So, $x = 2$ **and** $y = 0$

(viii) 3x – 4y = 4

x + 2y = 8

$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} = 3 \times 2 - 1 \times (-4)$

$= 6 + 4 = 10 \neq 0$

$A_x = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$

$|A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix} = 4 \times 2 - 8 \times (-4)$

$= 8 - 32 = 40$

$A_y = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix} = 3 \times 8 - 1 \times 4$

$$= \quad 24 - 4 \quad = \quad 20$$

$$x = \frac{|A_x|}{|A|} = \frac{40}{10} = 4$$

$$y = \frac{|A_y|}{|A|} = \frac{20}{10} = 2$$

So, $x = 4$ and $y = 2$

Q2. The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150 cm. Find the dimensions of the rectangle.

Solution:

(i) Method 1: Matrix Inversion Method:

Let length of rectangle is x cm and width of rectangle is y cm.

According to the given conditions

$x = 4y$

or $x - 4y = 0$ -------- (i)

Perimeter = 150cm

Perimeter = $2(x + y) = 150$

or $x + y = 75$ -------- (ii)

By solving (i) and (ii), we get

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

The coefficient matrix $M = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$ is non – singular because

$\det M = \begin{vmatrix} 1 & -4 \\ 1 & 1 \end{vmatrix} = 1 \times 1 - 1 \times (-4) = 1 + 4 = 5 \neq 0$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{M} \, Adj \, M \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 1 \times 0 + 4 \times 75 \\ (-1) \times 0 + 1 \times 75 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 0 + 300 \\ 0 + 75 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 300 \\ 75 \end{bmatrix}$$

$$= \begin{bmatrix} 60 \\ 15 \end{bmatrix}$$

$\Rightarrow$ $x = 60, \; y = 15$

So, length = x = 60 cm

width = y = 15 cm

(ii) Method 2: By Cramer's rule:

x – 4y = 4 ; x + y = 8

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -4 \\ 1 & 1 \end{vmatrix} = 1 \times 1 - (-4) \times 1$$

$$= 1 + 4 = 5 \neq 0$$

$$A_x = \begin{bmatrix} 0 & -4 \\ 75 & 1 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 0 & -4 \\ 75 & 1 \end{vmatrix} = 0 \times 1 - (-4) \times (75)$$

$$= 0 + 300 = 300$$

$$A_y = \begin{bmatrix} 1 & 0 \\ 1 & 75 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 1 & 0 \\ 1 & 75 \end{vmatrix} = 1 \times 75 - 0 \times 1$$

$$= 75 - 0 = 75$$

$$x = \frac{|A_x|}{|A|} = \frac{300}{5} = 60$$

$$y = \frac{|A_y|}{|A|} = \frac{75}{5} = 15$$

$\Rightarrow$ $x = 60,\ y = 15$

So, length = x = 60 cm

width = y = 15 cm

Q3. Two sides of a rectangle differ by 3.5 cm. Find the dimensions of the rectangle if its perimeter is 67 cm.

Solution:

(i) Method1: Matrix Inversion Method:

Let the length of the rectangle be **x** cm and its width be **y** cm.

According to the given conditions

$x - y = 3.5$

or $10x - 10y = 35$ (Hint: multiply both sides by 10)

and $2(x + y) = 2x + 2y = 67$

or $2x + 2y = 67$

$$\begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

The coefficient matrix M = $\begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}$ is non-singular because det M = $\begin{vmatrix} 10 & -10 \\ 2 & 2 \end{vmatrix} = 10 \times 2 - (-10) \times 2$

$= 20 + 20 = 40 \neq 0$

So, M is not a singular matrix

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1} \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{Adj\,M}{|M|} \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 2 & 10 \\ -2 & 10 \end{bmatrix} \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 2 \times 35 + 10 \times 67 \\ -2 \times 35 + 10 \times 67 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 70 + 670 \\ -70 + 670 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 740 \\ 600 \end{bmatrix} = \begin{bmatrix} 18.5 \\ 15 \end{bmatrix}$$

$\Rightarrow$ $x = 18.5,\ y = 15$

So, length is x = 18.5 cm and

width is y = 15 cm.

(ii) Method 2: By Cramer's rule:

10x – 10y = 35 ; 2x + 2y = 67

$$\begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35 \\ 67 \end{bmatrix}$$

$$A = \begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 10 & -10 \\ 2 & 2 \end{vmatrix} = 10 \times 2 - (-10) \times 2$$

$$= 20 + 20 = 40 \neq 0$$

$$A_x = \begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 10 & -10 \\ 2 & 2 \end{vmatrix}$$

$$= 35 \times 2 - (-10) \times 67$$

$$= 70 + 670 = 740$$

$$A_y = \begin{bmatrix} 10 & -10 \\ 2 & 2 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 10 & -10 \\ 2 & 2 \end{vmatrix} = 10 \times 67 - 35 \times 2$$

$$= 670 - 70 = 600$$

$$x = \frac{|A_x|}{|A|} = \frac{740}{40} = 18.5$$

$$y = \frac{|A_y|}{|A|} = \frac{600}{40} = 15$$

$\Rightarrow$ $x = 18.5,\ y = 15$

So, length is x = 18.5 cm and

width is y = 15 cm.

Q4. The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle.

Solution:

(i) Method1: Matrix Inversion Method:

Let each equal angle be x° and the third angle be y°

$\therefore$ $2x - 16 = y$

or $2x - y = 16$

and $2x + y = 180$ (Because sum of all the sides of a triangle is 180°)

$\therefore \quad \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$

The coefficient matrix M = $\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 2 = 2 + 2 = 4 \neq 0$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{M}\, Adj\, M \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1 \times 16 + 1 \times 180 \\ 1 \times 16 + 1 \times 180 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 16 + 180 \\ -32 + 360 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 197 \\ 328 \end{bmatrix} = \begin{bmatrix} 49 \\ 82 \end{bmatrix}$$

$\Rightarrow \quad x = 49,\ y = 82$

x + y + z = 180°

49° + 82° + z = 180°

z = 180°- 49°- 82° = 49°

So, the angles are 49°, 49°, 82°

(ii) Method 2: By Cramer's rule:

2x –y = 16 ; 2x + y= 180

$$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 2$$

$$= 2 + 2 = 4 \neq 0$$

$$A_x = \begin{bmatrix} 16 & -1 \\ 80 & 1 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 16 & -1 \\ 180 & 1 \end{vmatrix}$$

$$= 16 \times 1 - (-1) \times 180$$

$$|A_x| = 16 + 180 = 196$$

$$A_y = \begin{bmatrix} 2 & 16 \\ 2 & 80 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 2 & 16 \\ 2 & 80 \end{vmatrix} = 2 \times 180 - 16 \times 2$$

$$|A_y| = 360 - 32 = 328$$

$$x = \frac{|A_x|}{|A|} = \frac{196}{4} = 49$$

$$y = \frac{|A_y|}{|A|} = \frac{328}{4} = 82$$

$\Rightarrow$ $x = 49,\ y = 82$

x + y + z = 180°

49° + 82° + z = 180°

z = 180°- 49°- 82° = 49°

So, the angles are 49°, 49°, 82°

Q5. One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle.

Solution:

(i) Method1: Matrix Inversion Method:

Let two angles be x° and y°

$\therefore$ $2x + 12 = y$

or $2x - y = -12$

and $x + y = 90$ (sum of angles)

$\therefore$ $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$

The coefficient matrix M = $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ is non-singular because

det M = $\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 1 = 2 + 1 = 3 \neq 0$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{M}\, Adj\, M\begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 \times (-12) + 1 \times 90 \\ -1 \times (-12) + 2 \times 90 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} -12 + 90 \\ 12 + 180 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 78 \\ 192 \end{bmatrix} = \begin{bmatrix} 26 \\ 64 \end{bmatrix}$$

$\Rightarrow \quad x = 26, \ y = 64$

So, the angles are 26° and 64°

(ii) Method 2: By Cramer's rule:

2x –y = 12 ; x + y= 90

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 \times 1 - (-1) \times 1$$

$$= 2 + 1 = 3 \neq 0$$

$$A_x = \begin{bmatrix} -12 & -1 \\ 90 & 1 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} -12 & -1 \\ 90 & 1 \end{vmatrix}$$

$$= (-12) \times 1 - (-1) \times 90$$

$$|A_x| = -12 + 90 = 78$$

$$A_y = \begin{bmatrix} 90 & 12 \\ -1 & 2 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 90 & 12 \\ -1 & 2 \end{vmatrix}$$

$$= 90 \times 2 - 12 \times (-1)$$

$$|A_y| = 180 + 12 = 192$$

$$x = \frac{|A_x|}{|A|} = \frac{78}{3} = 26$$

$$y = \frac{|A_y|}{|A|} = \frac{192}{3} = 64$$

$\Rightarrow \quad x = 26, \; y = 64$

So, the angles are 26° and 64°

Q6. Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

(i) Method 1: Matrix Inversion Method:

Let the speeds of two cars be x km/h and y km/h

$x - y = 6$ -------------------- (i)

Distance covered in $4\frac{1}{2}$ hours = 600 – 123 = 477 km

$$4\tfrac{1}{2}x + 4\tfrac{1}{2}y = 477$$

$$\tfrac{9}{2}x + \tfrac{9}{2}y = 477$$

or $\frac{x}{2} + \frac{y}{2} = 53$

or $x + y = 106$ -------------------- (ii)

The two equations are (i) and (ii)

$$\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}6\\106\end{bmatrix}$$

The coefficient matrix M = $\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$ is non – singular because

det M = $\begin{vmatrix}1 & -1\\1 & 1\end{vmatrix} = 1\times 1 - 1\times(-1) = 1+1 = 2 \neq 0$

$$\begin{bmatrix}x\\y\end{bmatrix} = M^{-1}\begin{bmatrix}6\\106\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{M}\,Adj\,M\begin{bmatrix}6\\106\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}\begin{bmatrix}6\\106\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1\times 6+1\times 106\\-1\times 6+1\times 106\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}6+106\\-6+106\end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix}112\\100\end{bmatrix} = \begin{bmatrix}56\\50\end{bmatrix}$$

$\Rightarrow$ $x = 56,\ y = 50$

The speeds of the two cars are 56 km/h and 50 km/h

(ii) Method 2: By Cramer's rule:

$x - y = 6$; $x + y = 106$

$$\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}6\\106\end{bmatrix}$$

$$A = \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$$

$$|A| = \begin{vmatrix}1 & -1\\1 & 1\end{vmatrix} = 1\times 1 - 1\times(-1)$$

$$= \quad 1+1 \quad = \quad 2 \quad \neq \quad 0$$

$$A_x = \begin{bmatrix} 6 & -1 \\ 106 & 1 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 6 & -1 \\ 106 & 1 \end{vmatrix}$$

$$= 6 \times 1 - (-1) \times (106)$$

$$= 6 + 106 \quad = \quad 112$$

$$A_y = \begin{bmatrix} 1 & 6 \\ 1 & 106 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 1 & 6 \\ 1 & 106 \end{vmatrix} = 1 \times 106 - 6 \times 1$$

$$|A_y| = 106 - 6 \quad = \quad 100$$

$$x = \frac{|A_x|}{|A|} = \frac{112}{2} = 56$$

$$y = \frac{|A_y|}{|A|} = \frac{100}{5} = 50$$

$\Rightarrow$ $x = 56,\ y = 50$

The speeds of the two cars are 56 km/h and 50 km/h